The Steiner Model of Peak-Load Pricing

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Summary

This paper studies how robust or sensitive Steiner's peak load pricing results are to changes in certain assumptions. The peak-load problem deals with choosing the optimal pricing scheme leading to optimal output when there is a non-storable good whose demand fluctuates periodically at a uniform price. In the long-run planning point of view the problem also deals with optimal capacity of the system as opposed to short-run where the existing capacity is fixed an thus not subject to determination. I consider the peak-load problem for electric utilities.

Steiner's Peak load pricing results involves charging different prices for electricity in different time intervals aiming to mitigate the inefficiency of underutilized capacity over the cycle. The prices advocated by Steiner are set in accordance with long-run marginal costs and peak users bear all of the capacity cost. Relatively high price in the peak-demand period reduces the peak-demand and thus also the need for capacity investments for meeting peak demands. Comparatively, low price in the off-peak period is charged encouraging demand thereby making better use of existing capacity. Optimal capacity for the long-run planning point of view is simply found where capacity is equal to the peak load. However, this is strictly speaking not a result of the Steiner model – it is a relationship by definition.

I consider those papers relaxing one of the assumptions of the Steiner model without seriously undermining the insights from the basic model. I will study the implication of relaxing the assumptions: long-run planning point of view, linear costs, capacity fully variable in the long-run, independent demand, a welfare-maximizing social planner, single-technology and periods of equal lengths. I disregard those articles deriving a completely new model where the results are not directly comparable with the results of the Steiner model. I will not study the implication of changing the framework of a static, deterministic partial equilibrium model with exogenously determined demand functions, homogenous agents, no transmission costs, no intra period time varying demand, no storage possibilities, fully divisible capacity and no competitive element.

The result of price equal to long run marginal cost is not robust to changes in the assumption of long run planning point of view, linear costs, and fully variable capacity in the long run, dependent demand and the objective of maximizing welfare. When relaxing the assumptions,

prices still depend on marginal cost, however not the long-run marginal cost Steiner advocate but the short-run marginal cost. When considering a breakeven welfare-maximizing social planner or a profit-maximizing social planner, prices also depend on elasticity of demand. A breakeven constraint is imposed when there are non-linear costs or fixed capacity in the long run, to ensure the firm at least breaks even. If the monopoly is regulated the prices are also affected by the specific regulation. When dependent demand is considered, prices also depend on the cross-elasticity of demand. However, the relevance for this thesis of altering the assumption of independent demands is questionable. Dependent demand may violate the framework of partial equilibrium model and thus be outside the scope of this paper. With multiple technologies some sort of marginal cost pricing is still relevant. Price is set equal to the marginal cost of expanding the demand in that period.

Result of peak price higher than off-peak price follows automatically in the standard model in which there is a welfare objective, the firm has constant returns to scale in production and where capacity is fully variable in the long-run view. In general, nothing as strong as result 2 can be stated when considering a profit-maximizing monopoly or a breakeven welfare-maximizing social planner. Then prices depend on elasticity of demand and the pricing reversal phenomenon may occur depending on the parameters. Additionally, when we relax the assumption of independent demand, price will also depend on the cross-elasticity of demand and may contribute to pricing reversal.

Result of no responsibility for capacity cost imputed to those customers whose demand does not press upon capacity has been criticized on welfare grounds. Off-peak customers are also served by the capacity even if they do not press against the capacity limit. In the single-technology case the result is not valid for the short-run peak load problem, as capacity cost is only related to the long-run peak load problem. The one-technology (i.e. homogenous plant capacity) assumption is crucial for the result that peak users bear all of the capacity costs. When diverse technology is introduced off-peak customers are made to contribute to capacity costs, since they press against the capacity limit to the base-load capacity.

Result of optimal capacity found where it is equal to peak demand when optimized is relatively simple, and therefore does survive the different extensions reviewed. For all the extensions of the model, optimal capacity is equal to peak load demand due to the imputed capacity constraint. However, how to find optimal capacity in the extended models is different than Steiner advocate. Optimal capacity is found where the willingness to pay for an additional unit of capacity is equal to the cost of that unit including other components specific for the relaxed assumptions.

The robust result of Steiner's peak-load pricing when relaxing the above-mentioned assumptions is to set one price in each pricing period in accordance with the pattern of demand and prices are closely tied to variation in the marginal cost of generating electricity. Optimal capacity is equal to peak load.

Preface

When attending the course Environmental Economics (ECON4910), spring 2012, I came across a survey of the peak-load pricing literature by Michael A. Crew, Chitru S. Fernando and Paul K. Kleindorfer (1995). I was immediately fascinated by the theoretical development of peak-load pricing. Especially the numerous citations of an old paper written by Peter O. Steiner (1957) with the title: “Peak Loads and Efficient Pricing” raised my attention. It appeared to me that the development of the peak-load pricing literature until 1995 consisted of relaxing the assumptions of this simple Steiner Model. I was highly inspired and saw the chance of combining my interest for electricity economics, mathematics and history.

I deeply thank my supervisor Finn R. Førsund for his patient guidance, inspiration and insightful comments, and for sharing the fascination of the topic. I would also thank Nils C. Framstad for valuable explanation of line-integrals and uncertainty. A special thanks goes to Erika. Her constructive feedbacks have helped me organize my ideas properly.

At last, I would thank family and friends for their love and support in a time of both academic and personal challenges.

Any mistakes are mine alone.

Ragnhild Døble
Stavanger, May 2014

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Figures and tables

List of common variables

n	Number of periods in a cycle, $1,2 \dots i \dots n$
h	Number of technology types, $1,2 \dots l \dots h$
π	Profit function
TC	Total cost function
S	Consumer's surplus
x_i	Quantity of output supplied in period i
x_{li}	Quantity of output supplied in period i from technology/plant l
$p_i(x_i)$	Inverse demand function for output in period i
p_i	Market price for quantity x_i in period i
Q	Capacity of the system subject to determination (endogenous variable)
$\bar{Q}$	Capacity of the system not subject to determination (exogenous variable)
Q_l	Capacity of technology l
b	Operating costs per unit per period (marginal operating costs)
b_l	Operating costs per unit supplied by technology l per period.
β	The cost of providing a unit of capacity (marginal capacity cost)
β_l	The cost of providing a unit of capacity of technology l
λ_i	Shadow price on capacity constraint in period i
λ_{li}	Shadow price on capacity constraint for technology l in period i
γ	Shadow price on breakeven requirement
μ	Shadow price on regulatory constraint
κ	Ramsey number, $\kappa = \frac{\gamma}{1+\gamma}$
ε_i	Price elasticity, $\varepsilon_i = -\frac{p_i}{x_i}\frac{dx_i}{dp_i}$
ε_{ji}	Cross-price elasticity, $\varepsilon_{ji} = -\frac{p_j}{x_i}\frac{\partial x_i}{\partial p_j}, i \neq j$

1 Introduction

The peak-load problem is about finding the optimal pricing scheme leading to optimal output, and future optimal capacity when there is a non-storable good whose demand varies periodically. Real world pricing problems for electric utility (and telecommunications) motivated the early work on the peak load pricing theory. While peak-load problem was originally seen in connection to electric power and monopolies, its use have now spread to competitive industries such as hotels and airlines, and other public enterprises as postal services.

Steiner's paper *Peak Loads and Efficient* Pricing (1957) provided the basis for the peak-load pricing theory and is more cited than contemporary and subsequent peak-load pricing article (see appendix A). The peak-load pricing theory has progressively investigated the effects of relaxing the assumptions of the Steiner model to introduce a more complex and realistic framework. This paper study the assumptions of the Steiner model and discusses the implications of removing them. Beyond a short overview given in some papers as part of an introduction e.g. Abrate (2004), my research has not uncovered any paper consistently surveying how subsequent economist has developed the Steiner model.

This paper seeks to answer how robust or sensitive Steiner's peak load pricing results are to changes in certain assumptions, where robust is defined as the ability of an economic model to remain valid under different assumptions. Additionally, this paper examines if the results flow from a special crucial assumptions. Crucial assumption defined as, see Solow (1956, p. 65), one on which the conclusions do not depend sensitively.

As I am aiming for a unified framework I will limit the peak-load problem to the electric utilities, which was the original application of the peak-load literature. Due to their emphasis on precisely such a framework, I will focus on the peak-load problem literature from Steiner's article in 1957 to the survey of Crew, Fernando and Kleindorfer in 1995. The competitive element with the deregulation of the 1990s would change the model framework of the Steiner model and is not considered.

This paper considers those papers relaxing one of the assumptions of the Steiner model without seriously undermining the insights from the basic model. I disregard those articles deriving a completely new model where the results are not directly comparable with the results of the Steiner model.

I have only considered those models that are simple, which is related to Steiner's aim (1957, p. 604): to show the nature of the optimal solution to the peak load pricing problem "under some very restrictive assumption", which suggests he did not attempted to develop a model that could best mimic or approximate the reality. More likely he developed a model whose aim was to depict some ideal situation which could be used as a benchmark, or in the terminology of Gibbard and Varian (1978, p. 665), to derive a "caricature" who seek to "give an impression of some aspects of economic reality not by describing it directly, but rather by emphasizing – even to the point of distorting – certain selected aspect of the economic situations".

The paper is organized as follows. Section 2 provides a description of the peak load problem for the electricity sector. It will be explained why non-storability and periodic demand fluctuations give rise to the peak-load problem. Uncertainty and dynamics complicate the peak-load problem, however they will only be briefly explained. Because these features are not essential for the existence of the (theoretical) peak-load problem and studying them further is outside the scope of simple models. Section 3 describes the Steiner model and sets out the Steiner results. Section 4 reviews models where the following assumptions are altered:

1. The long-run planning point of view
2. Linear costs and capacity fully variable in the long-run
3. Independent demand
4. A welfare-maximizing social planner
5. Only one type of technology
6. Periods of equal length

I will only study their isolated effects.

The following assumptions – the fundament of the Steiner model – are unchanged: a static, deterministic partial equilibrium model with exogenously determined demand functions, homogenous agents, no transmission costs, no intra period time varying demand, no storage possibilities, fully divisible capacity and no competitive element. Relaxing these assumptions is too complicated to be treated carefully here. Finally, section 5 summarizes the implication for the Steiner results.

2 The Peak Load Problem

The peak-load problem deals with choosing the optimal pricing scheme for optimal output when there is a non-storable good whose demand fluctuates periodically (and stochastically) at a uniform price, which is a characterization of electric utility. Output refers to a unit of electricity. In the long-run planning point of view the problem also deals with optimal capacity of the system, as opposed to short-run point of view where the existing capacity is fixed an thus not subject to determination. The *systems capacity* of supplying electricity to customers consists of the plants generation capacity and the capacity of the transmission network connecting generators and consumers. The systems capacity refers to the maximum supply the system can provide without the line being damaged by the heat created due to resistance. For the rest of the article, capacity refers to the capacity of the system.

The issue with periodically varying demand of non-storable goods is the resulting underutilization of capacity over a cycle. A cycle (e.g. a day, week, season, year) is broken into multiple periods, where the peak period has the highest total demand of output and where the off-peak periods are the remaining periods. For electricity, demand and supply have to be in a continuous equilibrium to avoid power outage, thus capacity must be of such a size at least covering the peak-demand of the system. The peak demand dictates the size of generators, transmission lines and transformers even if peak demand is only for a small interval of time. Power outage occurs when peak demand exceeds the maximum supply levels that the electrical power industry can generate. Due to its specific nature, capacity cannot be varied to the extent that demand varies. Thus, peak demand requires the installation of additional capacity, which is under-utilized over the remainder of the cycle. Since capacity is not costless, the resulting idleness during the off peak is the basis for the peak-load problem and the motivation for pricing to mitigate this inefficiency.

An essential assumption for pricing being an effective instrument is that demand is not totally physical given but also depends on price. If the opposite were true, there would be "neither difficulty nor interest in the peak-load problem" (Steiner, 1957, p. 588). There would be no effect on output of any change in prices and thus one scheme of prices would be equivalent to any other. A natural assumption is therefore that prices may affect demand at any point in time, the higher price the lower demand and visa versa.

Andersson and Bohman (1985, p. 281) points out that since it takes several years to build a power plant, it becomes important to make a "clear distinction between investment rules to reach an efficient capacity for such a long term perspective and pricing rules for the short run, i.e. when capacity is given". The distinction between the peak-load pricing models where capacity is regarded fixed, and the peak load-pricing model where capacity is subject to determination will be central to the discussion in this paper.

Whether capacity is subject to determination or treated as fixed depends on the time point of view due to the long installation- and construction time. At the stage of the investment decision, future capacity could be regarded as fully variable. That is, capacity is variable and subject to determination in the long-run planning point of view. Notice that, investment decision about capacity is not at a marginal level but between a "definite, indivisible increases in capacity or non at all" (Andersson and Bohman, 1985, p. 281). However, capacity can be fixed in a long-term perspective. After the investment decision is made, the investments are (at least partially) irreversible and future capacity (partially) fixed. At any moment of time the existing capacity of the utility (at that time) is fixed. That is, capacity is fixed in the short run.

Periodically varying demand

The demand for electricity is subject to rapid variations over time of day, over the week and over seasons. Electricity use follows a daily cycle as well as a yearly cycle due to climatic change. This variation in demand is present even if price is the same over time. A peak demand for electricity may occur during daytime hours *at a given price* while demand slackens greatly during night-time of off-peak hours *at the same price.*

The total demand of a system can be described by a load duration curve, which gives information about the maximum demand of the system (the peak load) and its duration. A *load curve* order the systems demand data chronologically, while a *load duration curve* illustrates the demand data (kW) in descending order of magnitude, with the largest load to the left. The peak load is thus the maximum load of an electrical power-supply system at any point in time, literally the point at which the demand hits its peak. The hours with lowest consumption show the base load. In between is the shoulder or intermediate load. The off-peak load refers to the sum of base- and shoulder load. An example of a (smoothed) load- and a load duration curve is shown in figure 1. The area under the curve is the total usage of electricity during the cycle.

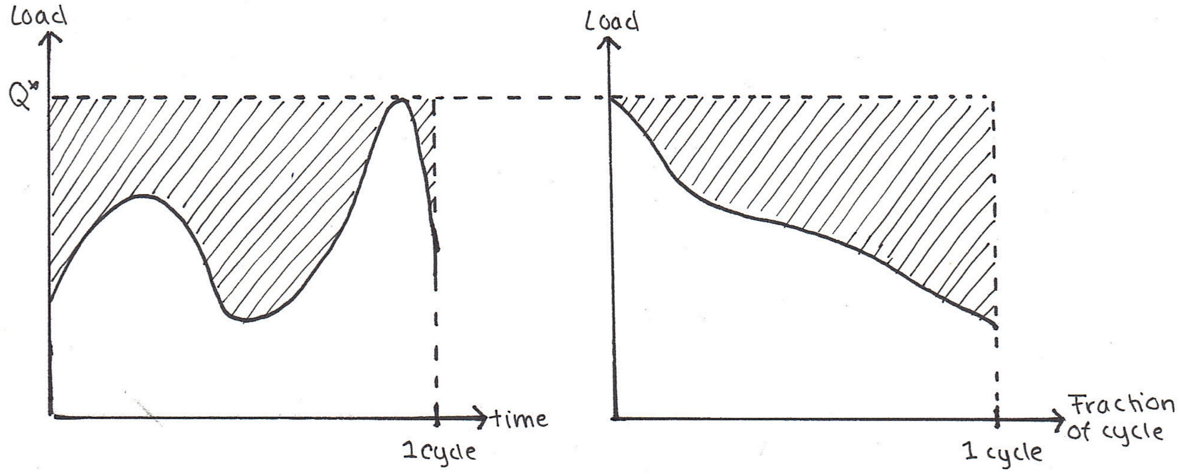

Figure 1: The load curve (left hand) and load duration curve (right hand)

To avoid power outage, capacity has to be greater than or equal to maximum demand at each point in time, and supply has to be in a continuous equilibrium to demand. To meet demand at the peak would require the installation of capacity Q^*, which is under-utilized over the remainder of the cycle. The shaded areas in figure 1 are the underutilized capacity over the cycle in question, and the area a pricing scheme searches to minimize. Non-storability and periodic demand fluctuations result in partial underutilization of the total capacity available in a cycle (shaded area), and this inefficiency is the core of the peak-load problem. The variation in demand is one of the underlying causes to peak load problem. If demand were constant over time (would be illustrated by a horizontal load- and load-duration curve) the capacity would be fully utilized over the cycle, and the peak load problem would not exist. Steiner (1957, p. 587) remarks that “a peak load problem will be said to exist at any price, if the quantities demanded in the two periods at that price are unequal”.

Non-storability

Unlike most products, electricity cannot be stored after it is generated; it must be generated at the time of demand. The non-storability serves as the other background to the existence of the peak-load problem; Lewis (1941a, p. 250) explains that if storage were possible the peak-load problem would be mitigated or not exists at all, as illustrated in figure 2. When storage is possible the capacity requirement is reduced to Q^{**}. Area B corresponds to pre-production in periods where demand for output is lower than the installed capacity can produce, and is saved to the excess demand in periods compromising area A. When A and B are equal the peak-load problem is fully mitigated if the cost of storage is insignificant. However, as describes by Nguyen (1976), if the cost of storage is not insignificant, the possibility of storage modifies, but does not eliminate the peak load problem. The peak- price will exceed off-peak price by the cost of storage. As e.g. noted by Mohring (1970, p. 693) the inability of storage implies that the production process should satisfy the demand in *real time*.

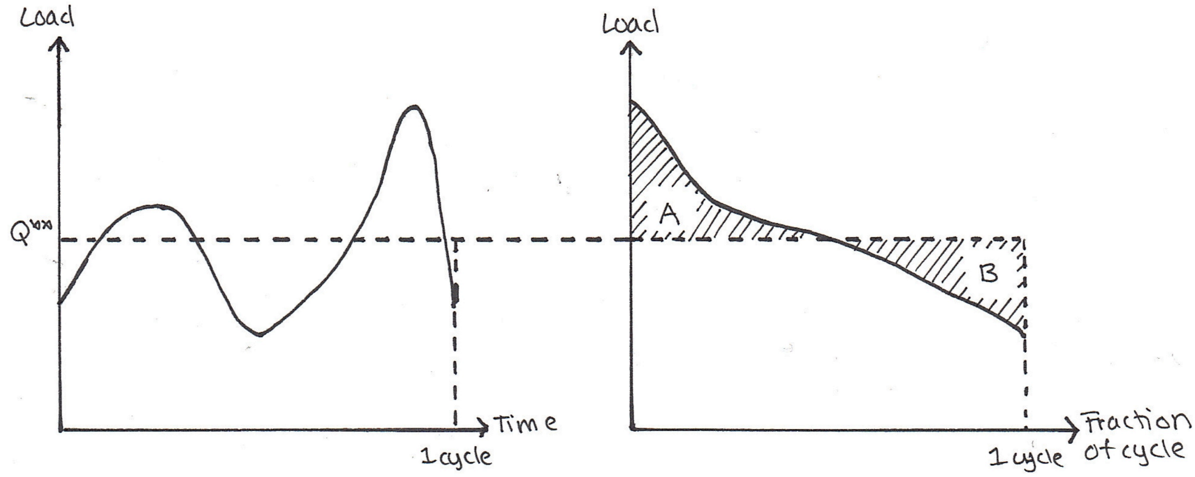

Figure 2: The load curve and load duration curve when storage is possible

Storability in the pure sense refers to pre-production, which is not possible with regard to electricity. (One exception is storage in batteries, though these may be considered as storage of chemical energy and not electricity). However, as pointed out by Gravelle (1976, p. 261, footnote 1) it is possible to *transform* electricity to other forms of power, and then back to electricity again at a later time. One example is regular hydropower reservoir, where water is stored in the reservoirs and transformed to electricity when water is released. Even if storage of electricity at some degree is possible, this is outside the scope of this paper and non-storability will be assumed throughout the paper[1]. The Steiner model is typically related to a system of conventional generating plants and not a system of hydropower where storage may be technological possible.

[1] For peak-load models where storage is possible see e.g. Gravelle (1976), Nugyen (1976) and Asbury and Mueller (1978).

Uncertainty and dynamics

In real life both the demand- and supply side is subject to uncertainty[2]. The demand of electricity is subject to random fluctuations due to the dependence on random components as weather and temperature among others. Whereas the supply of electricity depends on stochastic elements as broken lines, accidents, non-planned maintenance, plant breakdowns and component failure. Also the inputs for the different generations are random in nature, for solar power the sun is random, for wind power the wind is random, and for hydropower the rainfall is random. Kjølle et al (2007, p. 4) note that the uncertainty elements of the supply side have partially been mitigated by the introduction of revenue caps (the Cost of Energy Not Supplied arrangement[3]) in Norway 2001.

The real world is also characterised by dynamics. The world is such that demand and technology *changes* over time, so it is unrealistic to consider a static world. Demands may be subject to growth due to increasing population or changes consumption pattern. Technology may be improved over time such that capacity built in the future will be more cost efficient than the ones today, which will affect investment decisions.

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[2] For a peak-load model with uncertainty see e.g. Kleindorfer and Fernando (1993).

[3] The CENS arrangement is a model for incentive based regulation of supply quality, where the main objective is to give the network owners incentive to "plan, operate and maintain their networks in a socio-economic optimal way and thereby provide a socio-economic optimal level of reliability" (Kjølle, 2009).

3 Steiner's model and assumptions

Section 3.1 provides the graphics of the Steiner model. For pedagogic reasons a uniform pricing scheme assumed to cover total costs is used for comparison as done by Ault and Ekelund (1987, pp. 653-654) and Bergstrom and MacKie-Mason (1991). Section 3.2 derives a modern method of solving the Steiner's optimization problem. Section 3.3 list additional underlying assumptions of the model not mention by Steiner explicit. This section is based on comments and critiques by subsequent economists as well as my own. Section 3.4 summarize and decompose the peak-load pricing results, which the extended models in the following chapter will be compared to. Bailey (1972, p. 665) is the inspiration for the decomposition.

3.1 The Steiner Model

Steiner (1957, p. 585) seeks "to find an appropriate price policy that leads to the *correct amount of physical capacity* and its *efficient utilization*, and that also *covers the full social costs* of the resources used". It is evident that he only focuses on the long-run planning point of view, because capacity is to be determined: "our problem consists in determining the amount of capacity *ab initio*" (p. 588). It is assumed that the objective is to maximize social welfare in the market for electricity, and this assumption is implemented formally by maximizing the sum of consumers and producers' surplus.

Steiner considers a cycle divided in two-time periods of equal length, where his example is that of day and night. Fluctuating demand is captured by splitting the demand for output into two different periods, each with different known demand curves. It is further assumed that each demand curve being a declining function of the quantity of output in that period alone, that the two demand curves are not identical, are independent of each other, and that the demand curve for output in the first period lies everywhere above that in the second period. Where independent demand means that a price change in one period does not affect the demand in the other period.

Steiner makes the simple assumption about technology that only one type of plant is available of meeting demand. Let β be the per-cycle constant cost of providing a unit of capacity, and b is the constant operating cost of supplying a unit of output per period. Steiner further says:

“From the long-run (planning) point of view the marginal cost of a unit of output is thus b if there is excess capacity and $b + \beta$ if it requires new capacity”. This long-run marginal cost function is discontinuous, where the marginal cost jumps from b when there is excess capacity to $b + \beta$ when an additional unit of capacity is required to increase output.

Steiner’s aim is to determine optimal output x_i in each period i and the prices p_i that will lead buyers to purchase these quantities, and optimal capacity. Considering the determination of capacity, Steiner (1957, p. 587) says “the amount of capacity that is required is the maximum output in either period – that is, the maximum (or peak) demand on the system”. That is optimal capacity in the Steiner model is simply found where equal to the peak load.

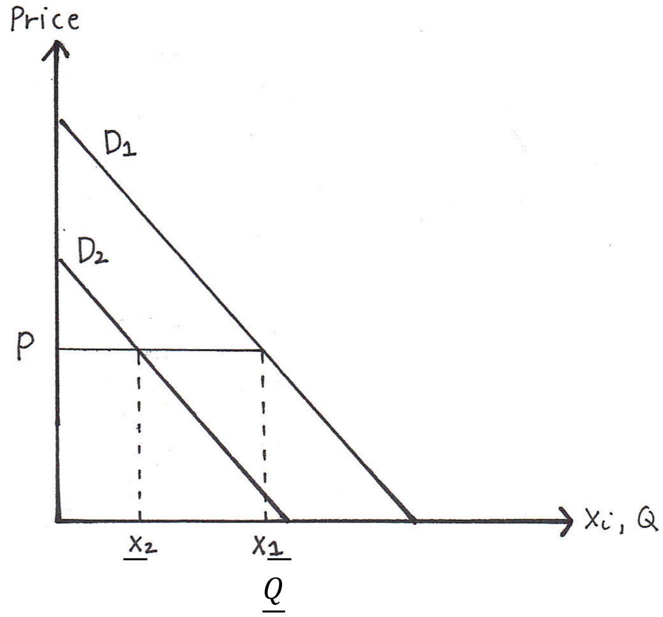

Figure 3: Uniform pricing

The two-period problem is given in figure 3, in which the demand curves for output in period 1 and 2 respectively, D_1 and D_2 are drawn. At any uniform price the demanded quantity in period 1 would be higher than for period 2, which follows from the assumption that the demand curve for period 1 lies everywhere above the demand curve for period 2. Hence, period 1 is the peak-load period and period 2 the off-peak period. An arbitrary uniform rate p would require a capacity of $\underline{Q} = \underline{x_1}$ units of capacity to serve the peak users. At rate p, off-peak demand would be $\underline{x_2}$ leaving $\underline{x_1} - \underline{x_2}$ units of capacity idle in the off-peak periods. As

3.1.1 The Firm Peak case

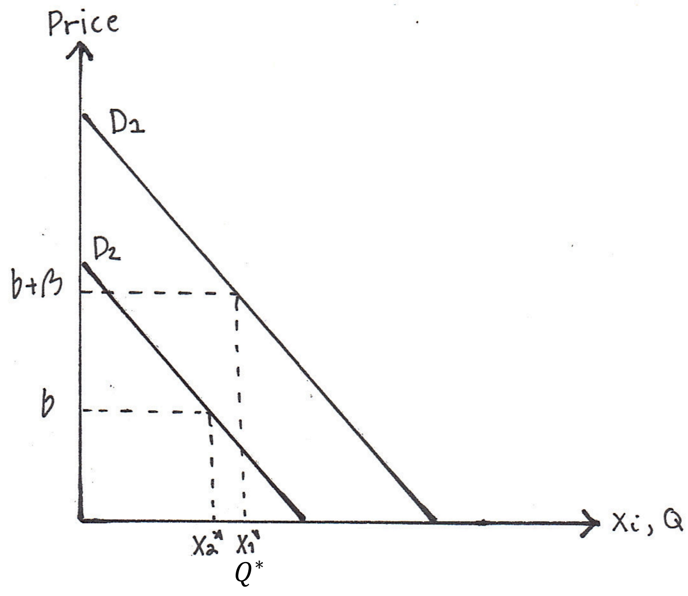

Figure 4: Optimal solution to the firm peak case

Given the demand curves, b and β in figure 4[4], Steiner argues that welfare optimality requires pricing at $p_1 = b + \beta$ and $p_2 = b$ leading to period outputs x_1^* and x_2^* and capacity $Q = x_1^*$. As Steiner (1957, p. 589) explains: "since the marginal capacity is that of users in period 1 only, the appropriate price for period 1 is $p_1 = b + \beta$. This capacity would be justified even if there were no demand in period 2. Hence period 2 users should be permitted to purchase output as long as they cover the operating costs of producing that (…) [output]".

Comparing the solution at prices $p_1 = b + \beta$ and $p_2 = b$ with the solution of some uniform pricing scheme, between b and $b + \beta$ assumed to cover total costs, we see that off-peak load is increased and peak load is reduced with the resulting better utilization of capacity in the off-peak period, and lower capacity requirements in the peak-period.

Figure 4 illustrate that the relationship between b and β is $b \approx \frac{1}{2}\beta$. Notice, that this relationship is dependent on the specific illustration in figure 4 and is not a general

[4] The reason for not reviewing the original graphical explanation of Steiner is due to his unconventional formulation of subtracting the operational costs b from the demand curves for output, which leads to at least two problems. First, the demand are then for "capacity" instead of output, which are misleading. Second, b is taken as the zero axes. This implies that b is erased from the graphics resulting in a pedagogical challenge when trying to connect his figures to his mathematical optimality results.

relationship between those two. Another relationship between b and β could easily have been illustrated.

According to the specific demand curves, operational cost b and capacity cost β in figure 4, pricing at $p_1 = b + \beta$ and $p_2 = b$ does not reverse the peak/off-peak pattern of period 1 being the peak load period. Thus, Steiner calls this case the *firm-peak case* (firm in the sense of unchanged relative to uniform prices), because period 1 is still the peak load period. Even though only running cost is charged in the off-peak period this is not sufficient for capacity to be utilized in the off-peak period and spare capacity always exists in the firm peak case. The pricing scheme $p_1 = b + \beta$ and $p_2 = b$ is the prices the extended models are to be compared to.

3.1.2 The Shifting Peak case

Additional to the firm-peak case Steiner also considers a case called the shifting-peak case, which is when the demand reaction of pricing at $p_1 = b + \beta$ and $p_2 = b$ *shift* the peak load period from being period 1 to period 2, and the pricing scheme above is no longer optimal. As explicit pointed out by Crew et al (1995, p. 221) this may be the case with another relationship between the demand curves or when capacity cost β in figure 4 is increased. The shifting-peak case has been subject to considerable attention in the early peak-load pricing literature (see Bye (1929), Lewis (1941b), Hirshleifer (1958), Williamson (1966), Gabor (1966) and Buchanan (1966)). However, as far as I can judge, the peak-load problem is exclusively related to the firm-peak case from the 1970s. The shifting peak case will not be considered as it is outside the scope of this paper.

A topic for further research would be to study the relevance of the shifting peak case and why it have achieved considerable attention in the early peak-load pricing theory and so little attention nowadays.

3.2 Mathematical background

Steiner (1957, p. 604) derives the following mathematical background for his analysis. Let $x_i \geq 0$ be quantity of output in period i and p_i be the corresponding market price for this quantity. I will follow the simplified approach of a two-period model throughout the thesis. Price and quantity are related by the function $p_i(x_i)$ which is the valuation of the marginal units in the respective periods, assumed continuous, differentiable and $p_i{}'(x_i) < 0\ \forall i$. Steiner (1957, p. 608) let $p_i = p_i(x_i)$ for all i. Steiner's original maximization problem is:

$$\max_{x_i}\left\{\sum_{i=1}^{2}\left(\int_0^{x_i} p_i\,(x_i{}')dx_i{}'\right) - \sum_{i=1}^{n} bx_i - \beta \max_i(x_i)\right\} \qquad i = 1,2 \qquad (1)$$

Where capacity Q is simply set equal to the peak load $Q = \max_i(x_i)$.

Before solving the maximization problem three comments are worth making. First, it is naturally to interpret $p_i(x_i)$ as the inverse demand curve for output in period i, which is the highest price the aggregated customers are willing to pay for different quantities of output. Second, the Steiner assumption "that the demand curve for output in the first period lies everywhere above that in the second period" can be formulated as: $p_1(x) > p_2(x)$ for all quantity of output x. Notice that Steiner in his original maximization problem used the expression $\sum_{i=1}^{n}\left(\int_0^{x_i} p_i\,(x_i)dx_i\right)$ instead of $\sum_{i=1}^{n}\left(\int_0^{x_i} p_i\,(x_i{}')dx_i{}'\right)$. The last formulation specify that the upper integral sign refers to a specific number and $x_i{}'$ to the independent variable in general.

Summarizing, the maximizing problem can be restated as:

$$\max_{x_i,Q}\left\{\sum_{i=1}^{2}\left(\int_0^{x_i} p_i\,(x_i{}')dx_i{}'\right) - \sum_{i=1}^{2} bx_i - \beta Q\right\}$$

subject to

$$\begin{gathered} p_i = p_i(x_i) \\ p_1(x) > p_2(x) \\ Q = \max_i(x_i) \\ x_i, Q > 0 \\ b, \beta\ given, i = 1,2 \end{gathered} \qquad (2)$$

Due to the specific formulation of the Steiner objective function, some initial assumption about x_1 and x_2 must be made to determine $\max_i(x_i)$. For the firm-peak case we have $x_1 > x_2$ *by definition*, this is in an assumption and not the result of the maximization problem. Thus, $\max_i(x_i) = x_1$and the total justified capacity is $Q = x_1$ while capacity is underutilized in the off-peak period $Q < x_2$, all by definition. The corresponding maximization problem to the assumption $x_1 > x_2$ could be stated as:

$$\max_{x_i}\left[\sum_{i=1}^{2}\left(\int_0^{x_i} p_i\,(x_i{}')dx_i{}'\right) - \sum_{i=1}^{2} bx_i - \beta x_1\right], i = 1,2 \tag{3}$$

We obtain the following result as Steiner does[5]

$$p_1 = b + \beta, \qquad p_2 = b \tag{4}$$

Peak price consist of marginal operating costs including cost of marginal capacity, while off-peak price is equal to marginal operating costs.

The breakeven property

Due to the linear operating and capacity costs the production process is characterized by constant returns to scale. For constant returns to scale, pricing at (4) will yield total revenues just sufficient to cover total costs. More specific, the profit function for the two-period firm peak case is given by:

$$\pi = \sum_{i=1}^{2} p_i x_i - \sum_{i=1}^{2} bx_i - \beta x_1 \tag{5}$$

Prices (4) inserted in (5) gives:

$$\pi = (\beta + b)x_1 + bx_2 - bx_1 - bx_2 - \beta x_1 = 0 \tag{6}$$

[5] Remembering that $\frac{d}{dx_i}\left(\int_0^{x_i} p_i\,(x_i{}')dx_i{}'\right) = p_i(x_i)$.

3.3 Comments

As noted by Crew et al (1995) the demand within each period is flat, so that in Steiner's case where there are two periods, say day and night, the demand is identical during each of the hours at day, and the same for the night[6]. When the demand within each period is flat and only one peak period and one off-peak period exists, the load curve and load duration curve are identical and stepwise instead of smooth. The assumption of peak demand existent for half of the cycle may be regarded as unrealistic, as it is more realistic to let the maximum demand of the system to only occur at a small amount of time. Gallant and Koenker (1984) notes that the two-period approach obscures some of the fine structure of the peak load problem: times at which local extrema occur, periods of extremely price sensitive demand, etc.

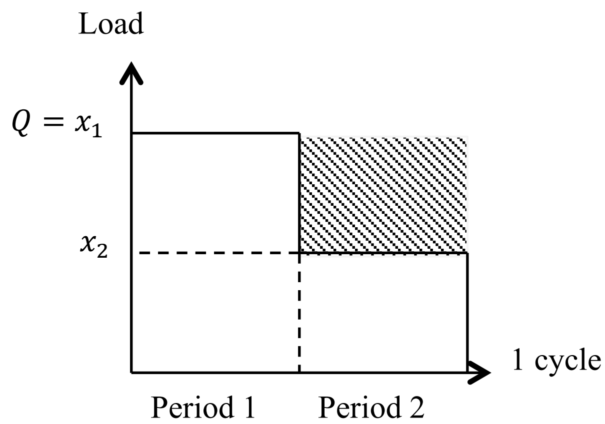

Figure 5: The load curve and load duration curve for the Steiner model

Bye (1929, p. 44, footnote 3) points out that when considering a demand curve we assume that production follow demands, not needs. The demand curves for each period and the peak period are exogenously given and the consumer's choices are not modelled (i.e. by using utility functions). That is, the model does not explain *why* the demand is higher in the first period than in the second. Period 1 is the peak-period by definition and not a result of the model.

Implicit in the Steiner model is the assumption of all power plants and consumers located in the same place, i.e. no transport/transmission costs, when in reality generators and consumers

[6] See Dansby (1978) for a model that relax the assumption of constant demand within each price period and allow for intraperiod time varying demand.

are spatially distributed[7]. In the Steiner model output and capacity is measured along the same axis implying that capacity can be divided into output units, i.e. there is an underlying assumption of fully divisible capacity as noted by Bailey (1972, p. 666). Andersson and Bohman (1985, p. 281) note that there is an underlying assumption of capacity in the long run being "continuously variable both when expanding and contracting it".

As noted by Turvey (1968, p. 103) and Lioukas (1983, p. 14) an implicit assumption of the Steiner model is that capacity is the same in all periods of the demand cycle. Steiner thus neglects that available capacity is not always equal to installed capacity, e.g. due to planned maintenance. Moreover, Turvey (1968, p. 102) notes that the capacity costs are pure peak-related. Underlying the formulation $\beta \max_i(x_i)$ in the welfare function is the assumption that off-peak customers are not charged the capacity costs even if capacity also serves the off-peak customers.

Crew et al (1995, p. 217) point out that in the welfare function (1) we find the assumption of equal valuation of benefit to the producers and consumers; the social planner is indifferent to the income redistribution effect. Baumol and Bradford (1970, p. 265) argue that the welfare-maximizing social planner may be a reasonable approximation of an economy in which all industry has been nationalized and in which the central planning agency is dedicated to the maximization of social welfare.

Williamson (1966, p. 811) notes that the Steiner model illustrate the first-best solution for the social planner and there is an underlying assumption of "all of the optimum conditions of production and exchange are satisfied elsewhere in the economy" because "some such assumption is necessary if we are to avoid second best digressions".

Underlying the Steiner model is also the assumption that is it technological possible and profitable to actually charge the marginal cost prices as involving one price for each period. This is an unreasonable assumption for bulk usage meters, which are only capable of measuring the amount of energy consumed but not the time at which is it used. However, for smart meter this is a more reasonable assumption. Joskow and Wolfram (2012) explains that

[7] See Arellano and Serra (2007) for a peak-load pricing model including transmission costs in a context of a two-node, two-technology electric power system.

a smart meter measure consumption of electricity in short intervals and automatically communicates that information back to the utility for billing purposes. The smart meter also receive information about electricity costs and give consumers information about their own usage. In this way customers will be able to manage their consumption. The Norwegian Water Resources and Energy Directorate (NVE, 2014) has informed that within 1st of January 2019 Smart meters should be in use in Norway. Whether end-use consumers actually face retail prices that reflect the variations in marginal generation costs depend on the metering technology available. There is also an underlying assumption that the consumers know the prices, i.e. prices set ex ante, when in real life electricity (sport) market price is set ex-post.

The Steiner model is a partial equilibrium model, where the clearance of the market in question is obtained independently from prices and quantities in other markets, and the clearance does not affect prices and quantities in other markets. This makes analysis much simpler than in a general equilibrium model, which includes an entire economy. Weakness is of course that the interrelations to the rest of the economy are disregarded.

3.4 The Steiner peak-load pricing result

Peak load pricing involves charging different prices for electricity in different time intervals aiming to mitigate the inefficiency of underutilized capacity over the cycle. Relatively high price in the peak-demand period reduces the peak-demand and thus also the need for capacity investments. Comparatively, low price in the off-peak period is charged encouraging demand thereby making better use of capacity.

The peak-load pricing advocated by Steiner could be decomposed into 4 results:

> *Result 1:* Set one price in each pricing period in accordance with the pattern of demand and long-run marginal cost of generating electricity.
>
> (Where *one price* means that any multiple tariff is not considered, and where *patterns of demand* refers to that any customer in a particular hour be treated the same regardless on how much he consumes.)

Result 2: Charge high prices when consumption tends to rise above the level of the capacity to discouraging use of electricity, and charge lower prices in periods with excess capacity to encourage demand.

Result 3: Allocate capacity cost only to those customers who contribute to additional capacity needs. Thus, the peak users bear all of the capacity costs and no responsibility for capacity cost is imputed to the off-peak customers whose aggregated demand does not press upon capacity.

Result 4: Optimal capacity is simply found where capacity is equal to the demand in the peak-period.

(Strictly speaking result 4 is not a result from the Steiner model, it is a definition)

4 How later economists have developed the Steiner Model

4.1 The short-run solution and capacity constraint

Hirshleifer (1958) and Williamson (1966) separated the short run and long run aspect of the problem and thus provided a more general solution than Steiner, who only considers the long-run planning point of view. Section 4.1.1 reviews the mathematical formulation of a peak-load problem distinguishing between the short-run peak load problem where capacity is fixed, i.e. an exogenous variable denoted $\bar{Q}$, and the long-run peak-load problem where capacity is variable and subject to determination (planning point of view), i.e. and endogenous variable denoted Q. Williamson (1966) is the inspiration for this section. Instead of capacity initially set equal to the maximum demand. He uses a capacity constraint that restricts the amount supplied in any period to its capacity. Section 4.1.2 reviews the graphics consistent with the mathematical formulation in 4.1.1. Hirshleifer (1958, p. 456, footnote 9) is the inspiration for the illustrations. Section 4.1.3 compare the findings with the Steiner peak-load pricing results.

4.1.1 Mathematical formulation

The short-run solution

When capacity is fixed the only decision is of pricing optimality within the capacity constraint $\bar{Q}$. Assumed operation is profitable the short-run maximization problem is:

$$\max_{x_i}\left[\sum_{i=1}^{2}\int_0^{x_i} p_i\,(x_i{}')dx_i{}' - \sum_{i=1}^{2} bx_i\right]$$

subject to

$$\begin{gathered} p_i = p_i(x_i) \\ p_1(x) > p_2(x) \\ x_i \le \bar{Q} \\ x_i > 0 \\ b, \bar{Q}\ given, i = 1{,}2 \end{gathered} \qquad (7)$$

Notice that β is not part of the short-run problem because capacity is fixed. The installation of new capacity is only related to the long-run problem and therefore also the cost of new capacity β. The Lagrangian for the problem is:

$$L(x_i) = \sum_{i=1}^{2} \int_0^{x_i} p_i\,(x_i{}')dx_i{}' - \sum_{i=1}^{2} bx_i - \sum_{i=1}^{2} \lambda_i(x_i - \bar{Q})$$

Endogenous variables are x_i and λ_i $(i = 1{,}2)$. The necessary first-order conditions are:

$$\frac{\partial L(x_i)}{\partial x_i} = p_i - b - \lambda_i = 0 \tag{8}$$

$$\lambda_i \geq 0, [= 0\ for\ x_i < \bar{Q}] \tag{9}$$

where λ_i is the shadow price on capacity, which is the change in the objective function maximized with respect to x_i of a marginal increase in the capacity constraint $\bar{Q}$. The shadow price on capacity can be interpreted as the marginal valuation of (i.e. the willingness to pay for) one extra unit of capacity. If demanded output is equal to the maximum amount existing capacity can provide in period i $(x_i = \bar{Q})$ the willingness to pay for one extra unit of capacity in period i is positive $(\lambda_i > 0)$. (I disregard the mathematical possibility of zero willingness to pay when production is at the capacity limit.) If there already exists unused capacity for production in period i $(x_i < \bar{Q})$, the willingness to pay for one extra unit of capacity in that period is naturally zero $(\lambda_i = 0)$.

Generally, optimal prices leading to optimal output is found where the marginal valuation of output is equal to the sum of marginal operating cost and the shadow price on capacity.

$$p_i = b + \lambda_i \tag{10}$$

Specifically, when production is at its maximum in period 1 $(x_1 = \bar{Q})$ and there are unused capacity in period 2 $(x_2 < \bar{Q})$ we have $\lambda_2 = 0$ and $\lambda_1 > 0$, and the optimal solution for each period i given capacity $\bar{Q}$ is:

$$p_1 = b + \lambda_1, \quad p_2 = b \tag{11}$$

Williamson (1966, p. 813) points out that the relationship between λ_i and β *signals* the investment direction. If the willingness to pay for one additional unit of capacity is larger than the actual cost of installing an additional unit of capacity $(\lambda_i > \beta)$ then an expansion of a plant is signalled. If the opposite is true ($\lambda_i < \beta$), Williamson says that plant should be “retired” rather than renewed until equality between λ_i and β are restored. This is a misinterpretation because the relationship between benefit (λ_i) and cost (β) only signals whether to invest or not. Williamson does not specify what is meant by “retiring”, could e.g. either be to scrap of resell capacity. In either way, if applied to reality this is unreasonable since excess capacity may be used due to unforeseen changes and future increase in demand, and resell a plant require a second-hand market for capacity. Williamson’s statement is highly related to the assumption of a deterministic and static world. When the willingness to pay for one additional unit of capacity is smaller than the actual cost of installing an additional unit of capacity is simply that an expansion of plant is *not* signalled.

The long-run solution

The long-run maximization problem is formulated as:

$$\max_{x_i,Q}\left[\sum_{i=1}^{2}\int_0^{x_i} p_i\,(x_i')dx_i' - \sum_{i=1}^{2} bx_i - \beta Q\right]$$

subject to

$$\begin{gathered} p_i = p_i(x_i) \\ p_1(x) > p_2(x) \\ x_i \leq Q \\ x_i, Q > 0 \\ b, \beta given, i = 1,2 \end{gathered} \tag{12}$$

The corresponding Lagrangian is:

$$L(x_i, Q) = \sum_{i=1}^{2}\int_0^{x_i} p_i\,(x_i')dx_i' - \sum_{i=1}^{2} bx_i - \beta Q - \sum_{i=1}^{2}\lambda_i(x_i - Q)$$

Endogenous variables are x_i, Q and λ_i ($i = 1,2$). The necessary first-order conditions are:

$$\frac{\partial L(x_i, Q)}{\partial x_i} = p_i - b - \lambda_i = 0 \tag{13}$$

$$\frac{\partial L(x_i, Q)}{\partial Q} = -\beta + \sum_{i=1}^{2}\lambda_i = 0 \tag{14}$$

$$\lambda_i \geq 0, [= 0\ for\ x_i < Q] \tag{15}$$

According to equation (14) optimal capacity is found where the cost of an additional capacity unit β is offset by the sum of the marginal valuation of that capacity for both periods $\sum_{i=1}^{2} \lambda_i$. Prices are still set as for the short-run solution (see equation (10)), because at any moment in time capacity at that time is fixed.

Specifically, let $x_1 = Q$ and $x_2 < Q$, then $\lambda_2 = 0$ and $\lambda_1 > 0$. According to (14) optimal capacity is found where the marginal willingness to pay for capacity by customers in period 1 is equal to the cost of one extra unit of capacity, that is $\beta = \lambda_1$. Optimal prices when capacity is optimized is equal to the prices Steiner advocate: $p_1 = b + \beta$ and $p_2 = b$. I will refer to these prices as long-run prices, as they are optimal prices for a cycle in the long-run planning point of view where capacity is optimized, i.e. where $\beta = \lambda_1$.

4.1.2 Graphical representation

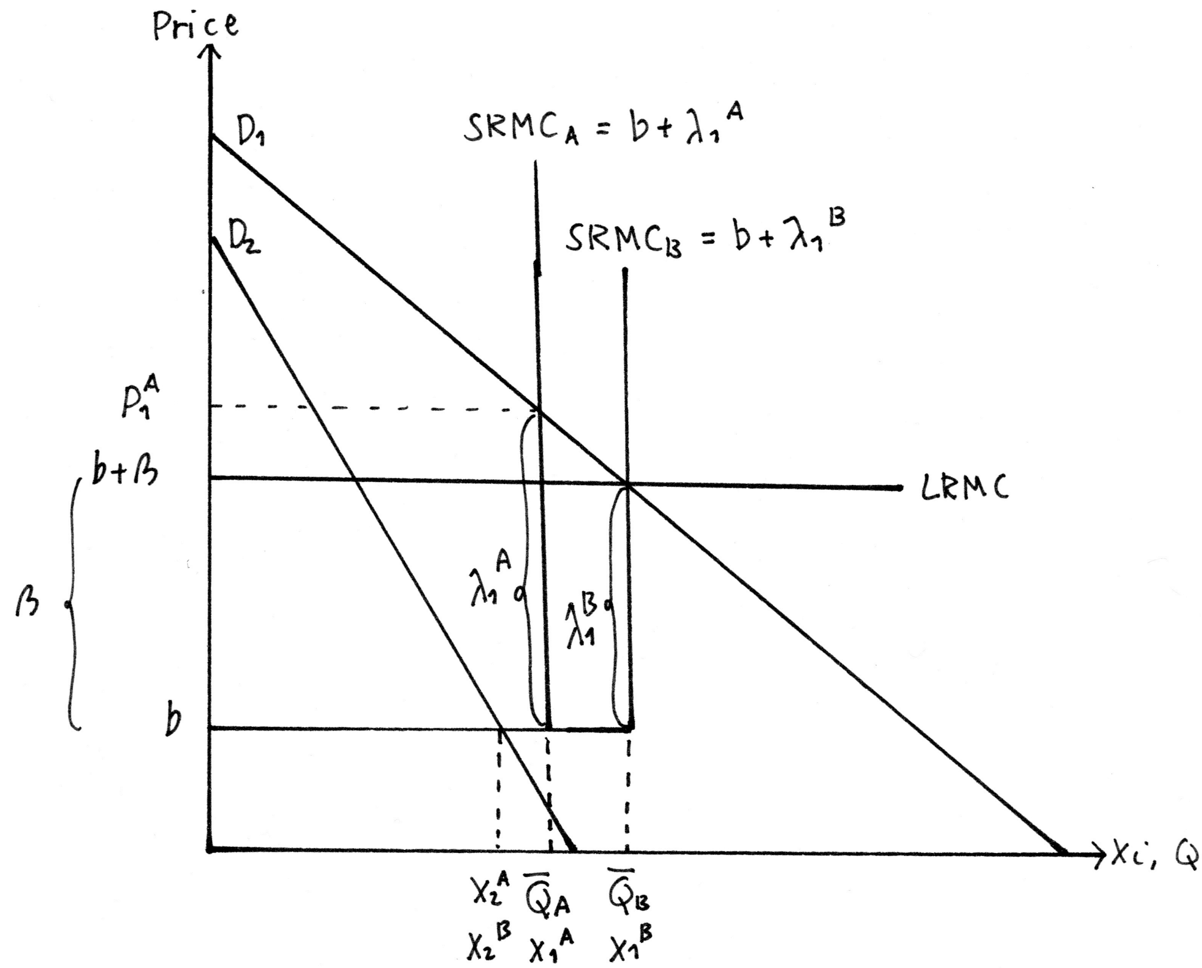

Figure 6: The short and long run solution

Figure 6 illustrates the short-and long run solution of the previous section 4.1.1. The marginal cost curve of meeting an additional unit of electricity consumption is different for the short- and long-run planning point of view[8]. The short-run marginal cost ($SRMC$) curve is the cost of meeting an additional unit of electricity consumption when capacity is fixed, and is derived from the short-run maximization problem (7).

$$SRMC = b + \lambda_i \tag{16}$$

The long-run marginal cost ($LRMC$) curve is the cost of providing an increase in consumption when optimal capacity adjustments are possible, and is derived from the long-run maximisation problem (12).

$$LRMC = b + \beta \tag{17}$$

Let $\bar{Q}_A$ be the existing capacity and assume $x_1 = \bar{Q}_A$ and $x_2 < \bar{Q}_A$. Because off-peak production is below the capacity limit the willingness to pay for an additional unit of capacity in period 2 is zero ($\lambda_2^A = 0$). Because the capacity level is reached in period 1 the willingness to pay for an additional unit of capacity is positive in period 1 ($\lambda_1^A > 0$). Optimal price for each period i to given capacity $\bar{Q}_A$ is found where the demand curve for each period crosses the curve $SRMC_A = b + \lambda_i^A$:

$$p_2^A = b, \qquad p_1^A = b + \lambda_1^A > b + \beta \tag{18}$$

The willingness to pay for an additional unit of capacity in period 1 (λ_1^A) is larger than the actual cost of installing a unit of capacity, signalling an expansion of the capacity (β). To ensure peak-demand is within the capacity limits a price sufficient higher than $b + \beta$ is charged.

For the long-run planning point of view optimal capacity is found where the willingness to pay for an additional unit of capacity equals the marginal cost of installing that unit, which is the case for capacity $\bar{Q}_B$ and its shadow price λ_1^B. After establishment of the optimal capacity, prices should be set as in the short run. The optimal short-run solution for given capacity $\bar{Q}_B$ is found where the demand curve for each period crosses the $SRMC_B = b + \lambda_i^B$:

$$p_2^B = b, \qquad p_1^B = b + \lambda_1^B = b + \beta \tag{19}$$

[8] See Andersson and Bohman (1985) for more about the short-and long-run marginal cost.

4.1.3 Implication for the Steiner result

Compared to the Steiner model two changes have occurred. First, the short-run aspect of the problem has been separated from the long run. The Steiner results are all related to the long-run planning point of view. Steiner does not discuss the short-run problem since he concentrates his attention upon the problem of installing and charging for capacity. Second, a capacity constraint has replaced the Steiner formulation of $Q = \max_i(x_i)$. The Steiner formulation requires an *initial* assumption about x_1 and x_2 to be able to solve the maximization problem (1) and information about the shadow prices on capacity is lost. When a capacity constraint is used, the relevant maximization problem is first solved then an assumption about x_1 and x_2 is made to get the specific prices for each period. This method ensures that the shadow price on capacity is included in the analysis.

These changes have had the following implications for the Steiner result. First, as the new method have shown, result 1 should be further specified that prices are to be set according to *short-run* marginal cost. The demand for electricity exhibits substantial variations over periods of time too short to permit capacity to be varied so as to keep price continuously equal to long-run marginal cost. For the short-run peak load problem result 3 of the Steiner model is no longer valid. Installing and charging for capacity is only related to the long-run peak load problem because capacity is fixed in the short run.

The use of a capacity constraint gives additional information about the capacity rule, result 4. In the Steiner model *optimal capacity* was simply found (or, strictly speaking – defined) where equal to demand in the peak period when prices were chosen optimally. With the new formulation of (12) *capacity* is still equal to the demand in the peak load period. However, *optimal capacity* is found according to equation (14), where the sum of marginal willingness to pay for one additional capacity unit over the periods is equal to the actual cost of the capacity unit.

4.2 The problem of deficits – The breakeven welfare-maximization model

Williamson (1966, p. 827) points out that if there are "increasing or decreasing returns to scale exists, or if capacity is given rather than subject to determination, optimal pricing will yield the zero net revenue result only accidentally if at all". Revenues are normally required to cover total costs despite the presence of fixed capacity or increasing economies of scale. Therefore a breakeven constraint is imposed (assumed a two-part tariff is not allowed) to ensure the firm at least breaks even in its operations:

$$\pi \geq 0$$

Crew et al (1995), Bailey and White (1974), Bailey (1972) and Pressman (1970) assumes that a monopolist supply electricity. The profit of a monopolist:

$$\pi = \sum_{i=1}^{2} p_i(x_i)x_i - TC$$

Section 4.2.1 explains why fixed capacity in the long-run point of view without any breakeven constraint will give zero net revenue only accidently. Thereafter, the breakeven constraint will be imposed and new pricing rules are introduced. Section 4.2.2 explains why non-linear costs may imply deficits in the absent of a breakeven constraint. Thereafter, the solution with a breakeven constraint is given using the cost function of Bailey and White (1974). Pressman (1970), Bailey (1972), Mohring (1970, pp. 696-698) and Bailey and White (1974) are the inspiration for section 4.2.2, and I will summarize their achievements in a unified framework. Section 4.2.3 reviews the implication for the Steiner results.

Notice, that the problem of deficits is related to the long-run peak load problem.

4.2.1 Capacity fixed in the long-run

Consider the long-run peak load problem where capacity for some reason is fixed in the long run. Capacity may be fixed e.g. due to irreversible past investment decisions, which still have to be paid for by the customers.

$$\operatorname*{Max}_{x_i}\left[\sum_{i=1}^{2}\int_{0}^{x_i} p_i\,(x_i')dx_i' - \sum_{i=1}^{2} bx_i - \beta\bar{Q}\right]$$

subject to

$$\begin{gathered} p_i = p_i(x_i) \\ p_1(x) > p_2(x) \\ x_i \leq \bar{Q} \\ x_i > 0 \\ b, \beta, \bar{Q}\ given, i = 1{,}2 \end{gathered} \tag{20}$$

The solution is equal to the short-run solution for fixed capacity (8) and (9). The profit to prices (8) and (9) when $x_1 = \bar{Q}$ and $x_2 < \bar{Q}$ is:

$$\pi = \sum_{i=1}^{2} p_i x_i - \sum_{i=1}^{2} bx_i - \beta\bar{Q} = \lambda_1 x_1 - \beta\bar{Q} = \bar{Q}(\lambda_1 - \beta)$$

Only when $\lambda_1 = \beta$ (i.e. when capacity is optimized) the monopoly breaks even in its operations. In the long-run *planning* point of view capacity could be fully adjusted so that optimum capacity happens and firm break even. For the long run when capacity is fixed $\lambda_1 = \beta$ occur only by accident, and the Steiner prices may not be optimal. Deficits occur when the monopolies supplying electricity is forced to fix the price at the marginal short-run cost, and the past irreversible investments customers have to pay for is larger than their willingness to pay ($\lambda_1 < \beta$).

Imposing the breakeven constraint

The long-run optimization problem to fixed capacity with the breakeven constraint imposed:

$$\max_{x_i}\left[\sum_{i=1}^{2}\int_{0}^{x_i} p_i\,(x_i')dx_i' - \sum_{i=1}^{2} bx_i - \beta\bar{Q}\right]$$

subject to

$$\begin{gathered} \pi = \sum_{i=1}^{2} p_i(x_i)x_i - \sum_{i=1}^{2} bx_i - \beta\bar{Q} \geq 0 \\ x_i \leq \bar{Q} \\ p_i = p_i(x_i) \\ p_1(x) > p_2(x) \\ x_i > 0 \\ b, \beta, \bar{Q}\ given, i = 1{,}2 \end{gathered} \tag{21}$$

The Lagrangian for the problem is:

$$L(x_i) = \sum_{i=1}^{2} \int_0^{x_i} p_i\,(x_i{}')dx_i{}' - \sum_{i=1}^{2} bx_i - \beta\bar{Q} - \sum_{i=1}^{2} \lambda_i(x_i - \bar{Q})$$
$$+ \gamma\left(\sum_{i=1}^{2} p_i(x_i)x_i - \sum_{i=1}^{2} bx_i - \beta\bar{Q}\right)$$

Endogenous variables are x_i, λ_i and γ $(i = 1{,}2)$. Where γ is the shadow price on the breakeven constraint, the reduction in the objective function maximized with respect to x_i of a marginally increase in profit requirement 0. If the profit requirement is increase (reduced) by 1-dollar then social welfare is reduces (increased) with γ dollars.

The necessary first-order conditions are:

$$\frac{\partial L(x_i)}{\partial x_i} = p_i - b - \lambda_i + \gamma\left(p_i(x_i) + \frac{\partial p_i(x_i)}{\partial x_i}x_i - b\right) = 0 \tag{22}$$

$$\lambda_i \geq 0, [= 0\ for\ x_i < \bar{Q}] \tag{23}$$

$$\gamma \geq 0, [= 0\ for\ \pi > 0] \tag{24}$$

The expression for optimal price to fixed capacity $\bar{Q}$ (22) could be rewritten as (se appendix B1):

$$p_i = \frac{b + \left(\frac{\lambda_i}{1+\gamma}\right)}{1 - \frac{\gamma}{1+\gamma}\left(\frac{1}{\varepsilon_i}\right)} \tag{25}$$

Where $\varepsilon_i = -\frac{p_i}{x_i}\frac{dx_i}{dp_i} > 0$ is the elasticity of the price of good i with respect to changes in its quantity. The price elasticity is the percentage reduction in demanded quantity for a good when price of that good is increased by one per cent. Compared to the first-best prices (10) prices are raised in inverse proportion to the absolute value of the price elasticity.

Specifically, when $x_1 = \bar{Q}$ and $x_2 < \bar{Q}$ (25) gives:

$$p_1 = \frac{b + \left(\frac{\lambda_1}{1+\gamma}\right)}{1 - \kappa\left(\frac{1}{\varepsilon_1}\right)}, \qquad p_2 = \frac{b}{1 - \kappa\left(\frac{1}{\varepsilon_2}\right)} \tag{26}$$

Where, according to Crew et al (1995, p. 219), $\kappa = \frac{\gamma}{1+\gamma}$ ($\kappa \in [0,1]$) is the so-called Ramsey number, which is positive except at the welfare optimum when the breakeven constraint is non-binding. The larger γ the larger is the Ramsey number, and the larger prices relative to the first-best prices (10). When the breakeven constraint is non-binding ($\pi > 0$) then $\gamma = 0$ implying $\kappa = 0$, and prices are equal to the first-best prices (10). When the breakeven constraint is binding ($\pi > 0$) then $\gamma > 0$ and the breakeven welfare-maximizing prices are larger than the first-best prices. The breakeven welfare-maximizing prices deviates the most from first-best prices (10) when $\gamma = \infty$, which implies $\kappa = 1$. (Control: $\lim_{\gamma \to \infty} \left[\frac{\gamma}{(1+\gamma)}\right] = 1$). As will be shown, when $\kappa = 1$ the prices for the breakeven welfare-maximizing social planner is identical to the prices for an unregulated profit-maximizing solution. The maximum reduction in social welfare of a 1-dollar increase in the profit requirement ($\gamma = \infty$) corresponds to the solution of an unregulated profit-maximising monopoly. Reducing the profit by 1-dollar will yield a substantial increase in social welfare.

This particular rewriting of peak-and off-peak prices when a breakeven constraint is imposed is found in Crew et al (1995, p. 225). For other rewritings see Bailey and White (1974, p. 78), Boiteux (1971, p. 235) or Bradford and Baumol (1970, p. 270).

4.2.2 Non-linear costs

In the Steiner model the total cost of producing x_1 and x_2 during period 1 and period 2 is clearly linear in production, ensuring the firm breaks even with marginal cost pricing. When we relax the assumption of linear cost function to a non-linear, the cost function may exhibit decreasing average costs, implying increasing returns, leading to deficits under marginal cost pricing. Pressman (1970), Bailey (1972) and Bailey and White (1974) consider a non-linear total cost function in their analysis.

Table 1: Overview over functions for total costs

	Total costs, TC
Steiner (1957)	$bx_1 + bx_2 + \beta Q$
Pressman (1970)	$D_1(x_1, Q) + D_2(x_2, Q) + K(Q)$
Bailey (1972)	$C(x_1, x_2) - \beta Q$
Bailey and White (1974)	$bx_1 + bx_2 + K(Q)$

Pressman (1970, p. 311) considers a non-linear total cost function where operating and capacity costs are not separable. The expression $D_i(x_i, Q)$ is the operating cost function for each period, and $K(Q)$ is the function for capacity cost. Pressman does not specify the first- and second order derivatives thus allowing both for increasing and decreasing returns to scale. Bailey (1972, p. 672) uses a cost function where operational- and capacity costs are separable. The function for operational costs are non-linear while the capacity cost function is linear. Derivatives are not specified.

Bailey and White (1974, pp. 78-80) use a total cost function where operating and capacity costs are also separable. Only the capacity costs are non-linear and they assume increasing returns to scale in capacity provision $K'(\mathrm{Q}) > 0, K''(\mathrm{Q}) < 0$. Notice that $K''(\mathrm{Q}) < 0$ may lead to convexity problems of the lagrangian. If the Lagrangian is concave in x_i and Q the solution according to the necessary first order conditions are sufficient for solving the maximization problem. When $K''(\mathrm{Q}) < 0$ the lagrangian is convex in Q and the maximization problem may not have an unique solution. Bailey and White do not mention this difficulty. I will use their cost-function in the following, assuming the solution from the necessary first order conditions are unique despite $K''(\mathrm{Q}) < 0$.

The maximization problem with non-linear capacity costs according to Bailey and White (1974) and no added constraints:

$$\max_{x_i, Q}\left[\sum_{i=1}^{2}\int_0^{x_i} p_i\,(x_i')dx_i' - \sum_{i=1}^{2} bx_i - K(Q)\right]$$

subject to

$$\begin{gathered} x_i \leq Q \\ p_i = p_i(x_i) \\ p_1(x) > p_2(x) \\ x_i, Q > 0 \\ b, \beta\ given, i = 1,2 \end{gathered} \tag{27}$$

The Lagrangian for the problem is:

$$L(x_i, Q) = \sum_{i=1}^{2}\int_0^{x_i} p_i\,(x_i')dx_i' - \sum_{i=1}^{2} bx_i - K(Q) - \sum_{i=1}^{2} \lambda_i(x_i - Q)$$

Endogenous variables are x_i, Q and λ_i ($i = 1,2$). The necessary first order conditions are:

$$\frac{\partial L(x_i, Q)}{\partial x_i} = p_i - b - \lambda_i = 0$$

$$\frac{\partial L(x_i, Q)}{\partial Q} = -\frac{\partial K}{\partial Q} + \sum_{i=1}^{2} \lambda_i = 0$$

$$\lambda_i \geq 0, [= 0 \; for \; x_i < Q]$$

When $x_1 = Q$ and $x_2 < Q$ optimal capacity is found where $\frac{\partial K}{\partial Q} = \lambda_1$ and optimal prices given optimal capacity is:

$$p_1 = b + \frac{\partial K}{\partial Q}, \qquad p_2 - b \tag{28}$$

Pricing according to (28) result in deficits because profit by marginal cost prices is:

$$\pi = Q\left(\frac{\partial K}{\partial Q} - \frac{K(Q)}{Q}\right)$$

The relationship between marginal capacity cost and average capacity costs determine whether the profit are positive, negative or zero. Increasing returns to scale imply decreasing average costs, which imply negative profits.

$$\frac{\partial}{\partial Q}\left(\frac{K(Q)}{Q}\right) < 0 \iff \frac{1}{Q}\left(\frac{\partial K}{\partial Q} - \frac{K(Q)}{Q}\right) < 0 \Rightarrow \frac{\partial K}{\partial Q} < \frac{K(Q)}{Q} \Rightarrow \pi < 0$$

Imposing the breakeven constraint

If a single price is all that can be levied for each type of customer, and if the firm is to break even the following maximization problem is appropriate:

$$\underset{x_i,Q}{\text{Max}}\left[\sum_{i=1}^{2}\int_0^{x_i} p_i\,(x_i{}')dx_i{}' - \sum_{i=1}^{2} bx_i - K(Q)\right]$$

subject to

$$\begin{gathered}\pi = \sum_{i=1}^{2} p_i(x_i)x_i - \sum_{i=1}^{2} bx_i - \beta Q \geq 0 \\ x_i \leq Q \\ p_i = p_i(x_i) \\ p_1(x) > p_2(x) \\ x_i, Q > 0 \\ b, \beta\ given, i = 1{,}2\end{gathered} \tag{29}$$

The Lagrangian for the problem is:

$$\begin{aligned}L(x_i, Q) = &\sum_{i=1}^{2}\int_0^{x_i} p_i\,(x_i{}')dx_i{}' - \sum_{i=1}^{2} bx_i - K(Q) - \sum_{i=1}^{2} \lambda_i(x_i - Q) \\ &+ \gamma\left(\sum_{i=1}^{2} p_i(x_i)x_i - \sum_{i=1}^{2} bx_i - K(Q)\right)\end{aligned}$$

Endogenous variables are x_i, Q, λ_i and γ ($i = 1{,}2$). The necessary first-order conditions are:

$$\frac{\partial L(x_i, Q)}{\partial x_i} = p_i - b - \lambda_i + \gamma\left(p_i(x_i) + \frac{\partial p_i(x_i)}{\partial x_i}x_i - b\right) = 0 \tag{30}$$

$$\frac{\partial L(x_i, Q)}{\partial Q} = -\frac{\partial K(Q)}{\partial Q} + \sum_{i=1}^{2} \lambda_i - \gamma\frac{\partial K(Q)}{\partial Q} = 0 \tag{31}$$

$$\lambda_i \geq 0, [= 0\ for\ x_i < Q] \tag{32}$$

$$\gamma \geq 0, [= 0\ for\ \pi > 0] \tag{33}$$

Equation (30) is the optimal short-run prices and as it is identical to equation (22) it may be rewritten according to (25). Equation (31) illustrates the optimal long-run capacity rule and may be rewritten as:

$$\sum_{i=1}^{2} \lambda_i = (1 + \gamma)\frac{\partial K(Q)}{\partial Q} \tag{34}$$

When $x_1 = Q$ and $x_2 < Q$ optimal capacity is found where $\lambda_1 = (1+\gamma)\frac{\partial K(Q)}{\partial Q}$ and optimal prices given optimal capacity is:

$$p_1 = \frac{b + \dfrac{\partial K(Q)}{\partial Q}}{1 - \kappa\left(\dfrac{1}{\varepsilon_1}\right)}, \qquad p_2 = \frac{b}{1 - \kappa\left(\dfrac{1}{\varepsilon_1}\right)} \tag{35}$$

Where, $\kappa = \frac{\gamma}{1+\gamma}$ ($\kappa \in [0,1]$) is the Ramsey number.

4.2.3 Implication for the Steiner result

Compared to the Steiner model two assumptions have been relaxed both leading to deficits. First, the Steiner assumption that capacity can be determined in the long run is relaxed. If capacity in the long run is fixed deficits may occur when the marginal capacity cost at the fixed capacity level is larger than the marginal willingness to pay for capacity in period 1. Second, when allowing for non-linear costs, deficits may occur due to increasing returns to scale. To ensure firm at least breaks even in its operations a breakeven constraint is imposed changing the optimal solution.

These changes have had the following implications for the Steiner result. The first-best solution of price equal to (short-run) marginal cost of generation capacity is not attainable due to deficits. Maximizing total welfare under the condition of non-negative profit is called Ramsey pricing. Ramsey pricing produces a second-best solution, since the efficiency condition of price equal marginal cost is lost, which violate result 1. Prices is no longer *only* set in accordance with pattern of demand and (short-run) marginal cost of generation capacity. Prices are also dependent on elasticity of demand if the breakeven constraint is binding ($\pi > 0$). The social welfare will be maximized not by setting price equal or even proportional to marginal cost, but where products with elastic demands are priced at levels close to the marginal cost, and visa versa (i.e. the inverse elasticity rule). Ramsey pricing minimizes the welfare loss when the monopoly supplying electricity are required to break even.

Notice also that the Steiner result 1 focus on one charge on peak users and one charge on off-peak users. An alternative to the breakeven pricing rules formulated above is the use of two-part or other non-linear tariffs; see for instance Lewis (1941a) or Bailey and White (1974, pp.

85-90), who introduce a model of peak-load pricing in which an entrance fee or customer charge as well as a peak and an off-peak usage charge is levied.

Result 2 of peak-price higher than off-peak price may be violated when prices also depend on elasticity of demand. Bailey and White (1974, p. 74) refer to this as the pricing reversal phenomenon. The pricing reversal will occur if the demand in the off-peak period is sufficiently more inelastic than that during the peak so as to compensate at the margin for the attribution of capacity costs to the peak period. The existence of the pricing reversal is of course an empirical question. However, nothing as general as result 2 can be stated when relaxing the assumption of variable capacity in the long run and linear costs to be able to charge the higher prices (35).

Result 4 about optimal capacity is extended when non-linear costs and breakeven constraint is imposed. Optimum capacity is found where the marginal willingness to pay for one additional unit of capacity in the peak-period is equal to the actual cost of installing one more unit of capacity multiplied with the shadow price on breakeven constraint. The optimum capacity level is reduced compared to the linear cost case.

4.3 Dependent demand

Steiner (1957, pp. 608-609) tries to extend the peak load problem to also include dependent demand by replacing $p_i(x_i)$ with $p_i(x_1, x_2)$ keeping everything else unchanged. The consumer's surplus:

$$S = \sum_{i=1}^{2} \left[\int_0^{x_i} p_i(x_1', x_2') dx_i' \right] - \sum_{i=1}^{2} p_i x_i \qquad (36)$$

Pressman (1970), who studies the peak-load problem with dependent demand, argues that this representation is inadequate because there is an underlying assumption of independent demand in the formulation of the consumer's surplus (36). The consumer's surplus used by Steiner is only valid when the demand for the ith good or service depends only on the quantity of that good or service, and *not* on any of the other n goods or service. The formulation of the consumer's surplus has to be adapted to the dependent demand property.

For the case of two-periods and two-good Pressman (1970) shows the more convenient formulation of multi-dimensional consumers' surplus with dependent demand by using a path-independent line integral of the form:

$$S = \int_{0,0}^{x_1,x_2} \left[\sum_{i=1}^{2} p_i(x_1', x_2')dx_i'\right] - \sum_{i=1}^{2} p_i x_i \tag{37}$$

Where the integration is being performed along some unspecified curve ("path") between the points $(x_1', x_2') = (0,0)$ and $(x_1', x_2') = (x_1, x_2)$. The formula (37) does not specify the path to be integrated along. According to the Gradient Theorem (also known as the fundamental theorem of calculus for line integrals) the line integral is only dependent on the end points – i.e. independent of the path between them – if and only if (p_1, p_2) is the gradient of some scalar function $F(x_1', x_2')$ and p_1 and p_2 are of class C^1.

$$\frac{\partial p_1}{\partial x_2} = \frac{\partial p_2}{\partial x_1} \tag{38}$$

Formula (38) is a necessary condition for the Gradient theorem to hold. The effect on peak period price resulting from a change in off-peak demand, is the same as the effect on off-peak period price resulting from a change in peak demand, that is when the income effects are negligible or zero. If (38) does not hold then (p_1, p_2) is no gradient of F resulting in path-dependent line integral. However, when (p_1, p_2) is the gradient of some scalar function $F(x_1', x_2')$:

$$\nabla F = \left(\frac{\partial F}{\partial x_1'}, \frac{\partial F}{\partial x_2'}\right) = (p_1, p_2)$$

Then, according to Young's Theorem, the cross-derivatives of F are equal:

$$\frac{\partial}{\partial x_2'}\left(\frac{\partial F}{\partial x_1'}\right) = \frac{\partial}{\partial x_1'}\left(\frac{\partial F}{\partial x_2'}\right) \Leftrightarrow \frac{\partial p_1}{\partial x_2'} = \frac{\partial p_2}{\partial x_1'}$$

Equation (38) follows when (p_1, p_2) is the gradient of some scalar function $F(x_1', x_2')$. Formula (38) is a sufficient condition for the Gradient theorem if p_1 and p_2 are of class C^1. When p_1 and p_2 are continuously differentiable then $\frac{\partial p_1}{\partial x_2'}$ and $\frac{\partial p_2}{\partial x_1'}$ exists (i.e. (38) exists).

The first derivative of the line integral with respect to x_i is equal to the price for that period:

$$\frac{\partial}{\partial x_i}\left(\int_{0,0}^{x_1,x_2}\left[\sum_{i=1}^{2} p_i(x_1{}', x_2{}')dx_i{}'\right]\right) = p_i$$

4.3.1 The model

Pressman (1970) analyse the general model:

$$\max_{x_i,Q}\int_{0,0}^{x_1,x_2}\left[\sum_{i=1}^{2}\int_{0}^{x_i} p_i\,(x_1', x_2')dx_i{}'\right] - TC$$

subject to

$$\begin{gathered}\pi = \sum_{i=1}^{2} p_i(x_1, x_2)x_i - TC \leq M \\ x_i \leq Q \\ p_i = p_i(x_i) \\ p_1(x) > p_2(x) \\ x_i, Q > 0 \\ b, \beta\ given, i = 1,2\end{gathered} \tag{39}$$

He considers the constraint $\pi \leq M$ instead of the breakeven constraint $\pi \geq 0$, where M is the maximum allowed profit for the firm. As far as I know, when a social planner maximizes welfare, $\pi \geq 0$ is the relevant constraint on profit and normally denoted a breakeven constraint. However, when the monopoly maximizes profit (see next chapter) $\pi \leq M$ is the relevant profit constraint, and normally denoted a *regulatory constraint*. Both the breakeven constraint and the regulatory constraints are subcases of profit constraints, where the first is related to a social planner maximizing welfare and the second to a profit-maximizing firm. Since Pressman considers a social planner that maximizes welfare the relevant profit constraint would be the breakeven constraint $\pi \geq 0$ and not the regulatory constraint $\pi \leq M$. The maximization problem with $TC = \sum_{i=1}^{2} bx_i + \beta Q$ is restated as:

$$\max_{x_i,Q} \int_{0,0}^{x_1,x_2} \left[\sum_{i=1}^{2} \int_{0}^{x_i} p_i\,(x_1', x_2')dx_i{}'\right] - \sum_{i=1}^{2} bx_i - \beta Q$$

subject to

$$\begin{gathered} \pi = \sum_{i=1}^{2} p_i(x_1, x_2)x_i - \sum_{i=1}^{2} bx_i - \beta Q \ \geq 0 \\ x_i \leq Q \\ p_i = p_i(x_i) \\ p_1(x) > p_2(x) \\ x_i, Q > 0 \\ b, \beta \ given, i = 1,2 \end{gathered} \tag{40}$$

The Lagrangian for the problem is:

$$L(x_i, Q) = \int_{0,0}^{x_1,x_2} \left[\sum_{i=1}^{2} \int_{0}^{x_i} p_i\,(x_1', x_2')dx_i{}'\right] - \sum_{i=1}^{2} bx_i - \beta Q) - \sum_{i=1}^{2} \lambda_i(x_i - Q)$$
$$+ \gamma\left(\sum_{i=1}^{2} p_i(x_1, x_2)x_i - \sum_{i=1}^{2} bx_i - \beta Q\right)$$

Endogenous variables are x_i, Q, λ_i and γ ($i = 1,2$). The necessary first-order conditions are:

$$\frac{\partial L}{\partial x_i} = p_i - b - \lambda_i - \gamma\left(b - \sum_{j=1}^{2} \frac{\partial p_j}{\partial x_i} x_j - p_i\right) = 0 \tag{41}$$

$$\frac{\partial L}{\partial Q} = -\beta + \sum_{i=1}^{2} \lambda_i - \gamma\beta = 0 \tag{42}$$

$$\lambda_i \geq 0, [= 0 \, for \, x_i < Q] \tag{43}$$

$$\gamma \geq 0, [= 0 \, for \, \pi > 0] \tag{44}$$

The optimum sized plant is derived form equation (42) and is similar to (31) ignoring the linearity of costs. The expression for optimal short-run price (41) could be rewritten as (see appendix B2):

$$p_i = \frac{b + \left(\frac{\lambda_i}{1+\gamma}\right)}{1 - \frac{\gamma}{1+\gamma}\left(\frac{1}{\varepsilon_i} + \frac{1}{\varepsilon_{ji}}\right)}\ , i \neq j \tag{45}$$

Where $\varepsilon_{ji} = -\frac{\partial x_i}{\partial p_j}\frac{p_j}{x_i}, i \neq j$ is the percentage change in demand for good i that occurs in a response to a percentage change in the price of the other god j, i.e. the cross-elasticity. The cross-elasticity is zero when demand is independent. Specifically, when $x_1 = Q$ and $x_2 < Q$ equation (45) gives:

$$p_1 = \frac{b + \left(\frac{\lambda_1}{1+\gamma}\right)}{1 - \kappa\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_{21}}\right)}, \qquad p_2 = \frac{b}{1 - \kappa\left(\frac{1}{\varepsilon_2} + \frac{1}{\varepsilon_{12}}\right)} \tag{46}$$

Where the Ramsey number $\kappa = \frac{\gamma}{1+\gamma}$, $\kappa \in [0,1]$. The short-run prices given optimized capacity, i.e. when $\lambda_1 = (1+\gamma)\,\beta$, is:

$$p_1 = \frac{b + \beta}{1 - \kappa\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_{21}}\right)}, \qquad p_2 = \frac{b}{1 - \kappa\left(\frac{1}{\varepsilon_2} + \frac{1}{\varepsilon_{12}}\right)} \tag{47}$$

4.3.2 Implication for the Steiner result

The Steiner result 1 is further extended to set prices in accordance also with the cross-price elasticity of demand, assumed the breakeven constraint is active. See section 4.3.2 for implication of result 2 when elasticity if demand is included in the analysis.

Whether it is necessary and reasonable to relax the assumption about independent demand is, of course, an empirical question. In circumstances where time patterns of consumption are relative inflexible, the independence assumption is presumably a close approximation. Additionally, including the dependence demand assumption transforms the partial equilibrium model of Steiner to a more general equilibrium model. We speak of a price change having impact one more than one market, when a price change in one period affects demand in the other period. The Steiner model clearly separate the market in period 1 and 2 and interdependencies may lie outside the scope of this paper because a more general model replaces the partial equilibrium model.

4.4 The profit-maximizing monopoly

The literature on peak load pricing essentially emerged in response to problems faced by most public utilities such as electricity supply industry, whose context lead the economists like Steiner to model pricing rules based on maximization of social welfare rather than profits. Boiteux (1971, p. 219) points out that the objective of maximizing social welfare calls for the monopolies to operate as though they were maximizing profits at constant prices. In the case of monopolies this requires substitution of their natural behaviour of equating marginal revenue to marginal cost, by the marginal cost pricing principle. However, the welfare-maximizing marginal-cost pricing rule may be inapplicable. Bailey (1972, p. 662) argues that the peak-load pricing model is (for the 1970s) most applicable to regulated monopolies, and therefore develops a peak-load pricing model concerned with the regulated profit-maximizing monopoly[9]. The profit maximizing monopoly without regulation is reviewed in Section 4.4.1 and with regulation in Section 4.4.2. Williamson (1974) are the inspiration for section 4.4.1. Bailey (1972) and Bailey and White (1974) are the inspiration for section 4.4.2. Section 4.4.3 analyse the implication for the Steiner result.

4.4.1 The unregulated profit-maximizing monopoly

$$\max_{x_i,Q}\left[\sum\nolimits_{i=1}^{2} p_i(x_i)x_i - \sum\nolimits_{i=1}^{2} bx_i - \beta Q\right]$$

subject to

$$\begin{gathered} x_i \le Q \\ p_i = p_i(x_i) \\ p_1(x) > p_2(x) \\ x_i, Q > 0 \\ b, \beta\ given, i = 1{,}2 \end{gathered} \tag{48}$$

The Lagrangian for the problem:

$$L(x_i, Q) = \sum\nolimits_{i=1}^{2} p_i(x_i)x_i - \sum\nolimits_{i=1}^{2} bx_i - \beta Q - \sum\nolimits_{i=1}^{2} \lambda_i(x_i - Q)$$

Endogenous variables are x_i, Q and λ_i $(i = 1{,}2)$. The necessary first-order conditions are:

$$\frac{\partial L(x_i, Q)}{\partial x_i} = p_i(x_i) + \frac{\partial p_i(x_i)}{\partial x_i}x_i - b - \lambda_i = 0 \tag{49}$$

$$\frac{\partial L(x_i, Q)}{\partial Q} = -\beta + \sum\nolimits_{i=1}^{2} \lambda_i = 0 \tag{50}$$

$$\lambda_i \ge 0, [= 0\ for\ x_i < Q] \tag{51}$$

[9] See Gerstner (1986) for a peak-load pricing model where the profit-maximizing firm operate in competitive markets.

The short-run pricing rule, equation (49), illustrates the natural behaviour of a monopoly of equating marginal revenue ($MR = p_i(x_i) + \frac{\partial p_i(x_i)}{\partial x_i} x_i$) to marginal short run cost ($b + \lambda_i$). Equation (49) could be rewritten as:

$$p_i = \frac{b + \lambda_i}{1 - \frac{1}{\varepsilon_i}} \tag{52}$$

Pricing according to (52) is called Ramsey pricing, where prices additionally to short-run marginal costs also depends on demand elasticities. The profit-maximizing firm take advantage of especially inelastic demand; the more inelastic demand (i.e. the smaller ε_i) the larger the price. To ensure positive prices we assume $\varepsilon_i > 1$. Except when $\varepsilon_i = \infty$ the profit-maximizing solution deviates from the welfare maximizing solution (10), which is the reason for the regulatory constrains.

When $x_1 = Q$ and $x_2 < Q$ optimum capacity is found where $\lambda_1 = \beta$. Optimum prices when capacity is optimized is:

$$p_1 = \frac{b + \beta}{1 - \frac{1}{\varepsilon_1}}, \qquad p_2 = \frac{b}{1 - \frac{1}{\varepsilon_2}} \tag{53}$$

Crew et al (1995, p. 219) and Bailey (1972, p. 674) points out that the Ramsey number denoted $\kappa = \frac{\gamma}{1+\gamma}$ provides a useful link between the welfare maximizing social planner subject to a breakeven constraint and the unregulated profit maximizing monopoly. The optimum prices for a breakeven social planner when capacity is optimized and linear costs:

$$p_1 = \frac{b + \beta}{1 - \kappa\left(\frac{1}{\varepsilon_1}\right)}, \qquad p_2 = \frac{b}{1 - \kappa\left(\frac{1}{\varepsilon_2}\right)} \tag{54}$$

When $\kappa = 1$ (i.e. $\gamma = \infty$) the solutions are identical.

4.4.2 The regulated profit-maximizing monopoly

Bailey (1972, p. 666) considers a regulatory limit, which holds profit at or below some level $F\ (x_1, x_2, Q)$. The regulatory limit will increase as the size of the system is increased, i.e. $\frac{\partial F}{\partial x_i}, \frac{\partial F}{\partial Q} > 0$. Assume that the monopolist is not allowed to include "useless" capacity in its rate base, but must actually put the investment to work in the form of operating capacity. This is a second-best solution for the monopoly as a revenue requirement is forced upon them.

$$\underset{x_i, Q}{\text{Max}}\left[\sum\nolimits_{i=1}^{2} p_i(x_i)x_i - \sum\nolimits_{i=1}^{2} bx_i - \beta Q\right]$$

subject to

$$\begin{gathered}
\pi = \sum\nolimits_{i=1}^{2} p_i(x_i)x_i - \sum\nolimits_{i=1}^{2} bx_i - \beta Q \leq F(x_1, x_2, Q) \\
x_i \leq Q \\
p_i = p_i(x_i) \\
p_1(x) > p_2(x) \\
x_i, Q > 0 \\
b, \beta\ given, i = 1{,}2
\end{gathered} \tag{55}$$

The Lagrangian for the problem is:

$$L(x_i, Q) = \sum\nolimits_{i=1}^{2} p_i(x_i)x_i - \sum\nolimits_{i=1}^{2} bx_i - \beta Q - \sum\nolimits_{i=1}^{2} \lambda_i(x_i - Q)$$
$$- \mu\left(\sum\nolimits_{i=1}^{2} p_i(x_i)x_i - \sum\nolimits_{i=1}^{2} bx_i - \beta Q - F(x_1, x_2, Q)\right)$$

Endogenous variables are x_i, Q, λ_i and γ $(i = 1{,}2)$. The necessary first-order conditions are:

$$\begin{aligned}
\frac{\partial L(x_i, Q)}{\partial x_i} &= p_i(x_i) + \frac{\partial p_i(x_i)}{\partial x_i}x_i - b - \lambda_i \\
&- \mu\left(p_i(x_i) + \frac{\partial p_i(x_i)}{\partial x_i}x_i - b - \frac{\partial F}{\partial x_i}\right) = 0
\end{aligned} \tag{56}$$

$$\frac{\partial L(x_i, Q)}{\partial Q} = -\beta + \sum\nolimits_{i=1}^{2} \lambda_i + \mu\left(\beta + \frac{\partial F}{\partial Q}\right) = 0 \tag{57}$$

$$\lambda_i \geq 0, [= 0\ for\ x_i < Q] \tag{58}$$

$$\mu \geq 0, [= 0\ for\ \pi < M] \tag{59}$$

Where μ is the shadow price on the profit ceiling F and is the change in the optimized objective of profit of relaxing the profit ceiling with one unit. The shadow price on the profit ceiling and may be interpreted as the monopolies marginal willingness to pay for relaxing the profit ceiling by one unit. The monopolies are not willing to pay more for relaxing the profit ceiling than the actual increase in profit ceiling, which would imply a marginal reduction in profit. That is μ lies between zero and one.

The short-run pricing rule, equation (49) could be rewritten as:

$$p_i = \frac{b + \frac{\lambda_i}{1-\mu} - \left(\frac{\mu}{1-\mu}\right)\frac{\partial F}{\partial x_i}}{1 - \frac{1}{\varepsilon_i}}, \varepsilon_i \neq 1, \mu \neq 1 \quad (60)$$

Specifically, when $x_1 = Q$ and $x_2 < Q$ the short-run solution is:

$$p_1 = \frac{b + \frac{\lambda_1}{1-\mu} - \left(\frac{\mu}{1-\mu}\right)\frac{\partial F}{\partial x_1}}{1 - \frac{1}{\varepsilon_1}}, \qquad p_2 = \frac{b - \left(\frac{\mu}{1-\mu}\right)\frac{\partial F}{\partial x_2}}{1 - \frac{1}{\varepsilon_2}} \quad (61)$$

The prices are reduced relative to the unregulated case (52). The short-run solution when capacity is optimal chosen, i.e. when $\lambda_1 = \beta\ (1-\mu) - \mu\frac{\partial F}{\partial Q}$, is:

$$p_1 = \frac{b + \beta - \left(\frac{\mu}{1-\mu}\right)\left(\frac{\partial F}{\partial x_1} + \frac{\partial F}{\partial Q}\right)}{1 - \frac{1}{\varepsilon_1}}, \qquad p_2 = \frac{b - \left(\frac{\mu}{1-\mu}\right)\frac{\partial F}{\partial x_2}}{1 - \frac{1}{\varepsilon_2}} \quad (62)$$

Regulation is effective in that, relative to the unregulated case, the monopoly profits and prices are reduced and capacity is increased. According to (62) peak prices is reduced more than off-peak price due to $\frac{\partial F}{\partial Q}$. Because constrained profits increase with the level of capacity, the firm passes on the benefits of regulation to those users whose increased demand will cause an increase in capacity (i.e. the peak-users) relative to the unregulated profit-maximizing monopoly. (Whether the level of capacity expands between the beyond the welfare-maximizing level depend on the parameters of the system, nothing general can be said.)

The method of regulation can have substantial effects on the distribution of price reductions between peak-and off-peak users. Bailey (1972) considers three particular forms for the profit ceiling F. The first is the rate of return-on-investment[10] constraint where the firm's earnings may not exceed a fixed amount s for each unit of its capacity, assumed $s > \beta$.

[10] See Crew and Kleindorfer (1981) for the effects of rate-of-return regulation under a diverse technology for a profit-maximizing monopolist.

$$F = (s - \beta)Q \tag{63}$$

The second method of regulation limits the firm's profits to a fixed amount q per unit of output.

$$F = q\sum_{i=1}^{2} x_i \tag{64}$$

The third method of constraint involves permitting the firm a fair return on total costs, $m > 0$.

$$F = m\left(\sum_{i=1}^{2} bx_i + \beta Q\right) \tag{65}$$

The solutions for the specific cases are:

Table 2: Particular forms for the profit ceiling

	Peak price, p_1	Off-peak price, p_2
Rate-of return regulation (63)	$\dfrac{b + \beta - \left(\frac{\mu}{1-\mu}\right)(s - \beta)}{1 - \frac{1}{\varepsilon_1}}$	$\dfrac{b}{1 - \frac{1}{\varepsilon_2}}$
Regulation limiting profit per unit (64)	$\dfrac{b + \beta - \left(\frac{\mu}{1-\mu}\right)q}{1 - \frac{1}{\varepsilon_1}}$	$\dfrac{b - \left(\frac{\mu}{1-\mu}\right)q}{1 - \frac{1}{\varepsilon_2}}$
Return on cost regulation (65)	$\dfrac{(b + \beta)\left(1 - m\left(\frac{\mu}{1-\mu}\right)\right)}{1 - \frac{1}{\varepsilon_1}}$	$\dfrac{b\left(1 - m\left(\frac{\mu}{1-\mu}\right)\right)}{1 - \frac{1}{\varepsilon_2}}$

4.4.3 Implication for the Steiner results

The Steiner result 1 must be further extended to include elasticity of demand when profit maximization is the objective. (Notice that this does not assume binding breakeven constraint a is the case for the breakeven welfare-maximization model.) Additionally, when the monopoly is subject to regulation prices additionally depend on the change in the profit ceiling of a marginal change in produces output $\frac{\partial F}{\partial x_i}$, and the shadow price on the profit ceiling F.

Result 2 may be altered when profit maximizing is the objective because prices depend on elasticity resulting in a possible pricing reversal. For all the specific cases of the profit ceiling F, whether peak price is larger than off peak price or the other way around depends on the relative size of b, β, the price elasticity's, and the specific parameters s, M and q.

Pricing reversal is most likely to happen for rate of return regulation. Rate of return regulation is asymmetric in its effects because the entire effect of the regulation is reflected in the peak price. Rate of return regulation gives an incentive to lower the rate to the peak users while keeping the off-peak rate at the monopoly level. Intuitively, because constrained profits increase with the level of capacity, the firm passes on the benefits of regulation to those users whose increased demand will cause an increase in capacity. Accordingly, the likelihood is increased of finding the pricing reversal. Regulation limiting profits per unit and return on cost regulation have symmetric effects on peak- and off-peak prices; both prices are reduces compared to the unregulated profit-maximization. The regulation limiting profit per unit is symmetric in that whether the operating costs arise in peak-or off-peak period is irrelevant. The return on cost regulation is symmetric in that it treats all dollar costs in the same way independent of whether they arise from operating or capacity considerations.

Result 4 about optimal capacity is extended when the monopoly is subject to regulation. Optimum capacity is found where the marginal willingness to pay for an additional unit of capacity over periods equal the marginal cost of expanding capacity minus an interaction effect between the capacity cost and the regulatory constraint.

4.5 Multiple Technologies

We now relax the assumption of a single-technology and allow for more than one technology with different cost characteristics. Crew and Kleindorfer (1971; 1975; 1976), Wenders (1976), Panzar (1976), and Turvey (1968) have studies this case. Section 4.5.1 reviews the optimal plant mix. Section 4.5.2 reviews the optimal prices. Both sections are mainly based on Crew and Kleindorfer (1971) and Crew et al (1995, pp. 221-225) who summarize their results. Section 4.5.3 describe the implications of allowing for multiple technologies on the Steiner results.

4.5.1 Optimal technology mix

Assume two plants available for meeting the demands. Plant 1 and plant 2 have constant operating costs of b_1 and b_2 per unit per period, and capacity costs of β_1 and β_2 per unit of capacity, respectively. Assume that the technologies have been numbered such that the following condition hold:

$$b_1 < b_2, \qquad \beta_1 > \beta_2 \tag{66}$$

If such a numbering were not the case, say, $b_1 > b_2, \beta_1 > \beta_2$, then plant 1 would be dominated by plant 2. For *both* technologies to be used in the optimal solution the following must hold:

$$\frac{\beta_1 - \beta_2}{2} < b_2 - b_1 < \beta_1 - \beta_2 \tag{67}$$

If the left-hand (respectively, right-hand) inequality is violated, only technology 2 (respectively, technology 1) need be used in any optimal solution. Using the numbering (66), the efficient technology frontier is downward sloping and convex in (b, β) space as shown in figure 7 for four types of technology.

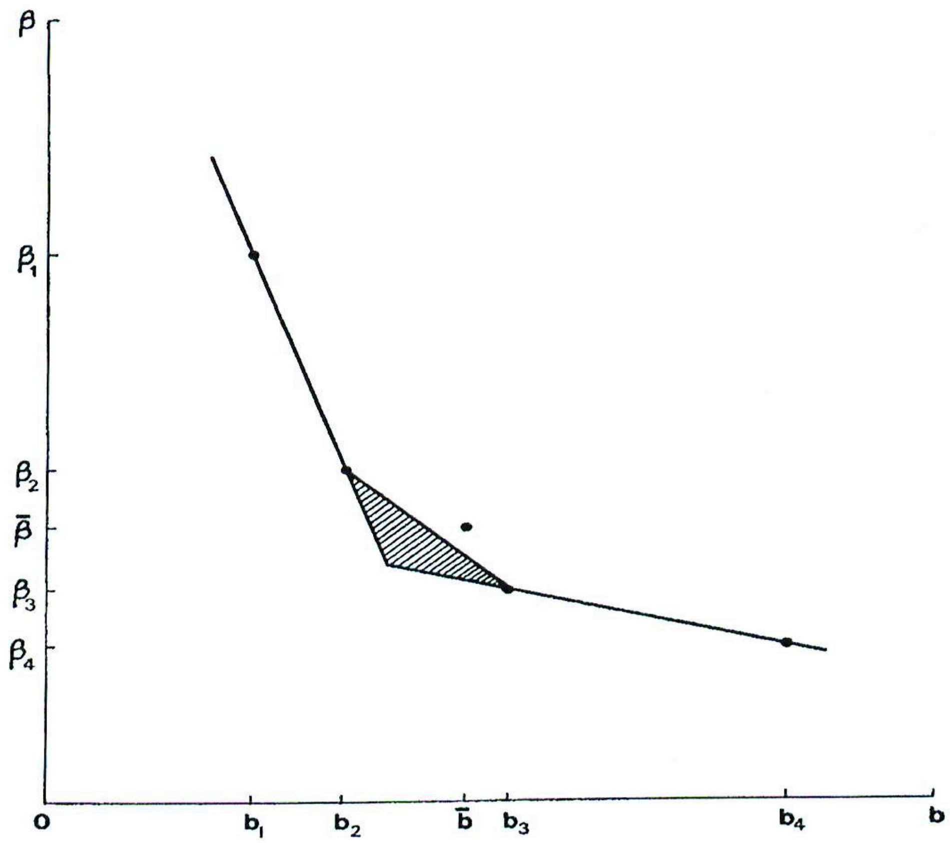

Figure 7: Efficient frontier. *Source: Crew and Kleindorfer (1975, p. 84)*

Notice, that plant type $(\bar{b}, \bar{\beta})$ would be ruled out in this case as its addition to the existing plant mix would violate the required convexity of the technological frontier. Some combination of technologies at the efficient frontier, would dominate any technology $(\bar{b}, \bar{\beta})$.

When having multiple technologies with different cost characteristics according to (66) and (67) it is economical to employ technology 1 with high capacity (i.e. construction) costs and low operating costs as base load technology, and use technology 2 with low capacity costs and high operating costs as peaking technology. That is technology 1 is continuously operated over the cycle, while technology 2 is used as a top-up in the peak period. This technology mix would offer cost advantages in meeting a peak demand of short duration. Therefore, capacity is installed in order of decreasing capacity costs, and once capacity is installed it should be operated in order of increasing operating costs (i.e. merit order).

4.5.2 Optimal pricing

Let $b_l \geq 0$ be the constant operating cost per unit of x_i per period supplied from plant $l = 1,2$. Let the quantity supplied from plant $l = 1,2$ in period $i = 1,2$ be denoted by x_{li}, and the capacity of plant $l = 1,2$ be denoted by Q_l. The systems capacity is $Q = \sum_{l=1}^{2} Q_l$. The problem to be maximized could be stated as[11]:

$$\max_{x_i, Q} \left[\sum_{i=1}^{2} \int_0^{x_i} p_i\,(x_i')dx_i' - \sum_{i=1}^{2} \sum_{l=1}^{2} b_l\, x_{li} - \sum_{l=1}^{2} \beta_l\, Q_l \right]$$

subject to

$$\begin{gathered} x_{1i} + x_{2i} = x_i \\ x_{li} \leq Q_l \\ p_i = p_i(x_i) \\ p_1(x) > p_2(x) \\ x_i, Q_l > 0 \\ b, \beta\ given, i = 1,2, l = 1,2 \end{gathered} \tag{68}$$

The Lagrangian for the problem is:

$$L(x_{li}, Q_l) = \sum_{i=1}^{2} \int_0^{x_{1i}+x_{2i}} p_i(x_i)dx_i - \sum_{i=1}^{2} \sum_{l=1}^{2} b_l\, x_{li} - \sum_{l=1}^{2} \beta_l\, Q_l - \sum_{i=1}^{2} \sum_{l=1}^{2} \lambda_{li}(x_{li} - Q_l)$$

[11] See Crew and Kleindorfer (1971, pp. 1375-1376) for a profit-maximizing monopoly with diverse technology.

Endogenous variables are x_{li}, Q_l and λ_i ($i = 1,2, l = 1,2$). The necessary first-order conditions are:

$$\frac{\partial L}{\partial x_{li}} = p_i - b_l - \lambda_{li} \leq 0[= 0\, for\, x_{li} > 0] \tag{69}$$

$$\frac{\partial L}{\partial Q_l} = -\beta_l + \sum_{i=1}^{2} \lambda_{li} = 0 \tag{70}$$

$$\lambda_{li} \geq 0\ [= 0\, for\, x_{li} < Q_l\ ] \tag{71}$$

The Lagrangian parameter λ_{li} is the shadow price of a marginal increase in Q_l, and can be interpreted as the willingness to pay for an additional unit of capacity of technology l by customer's in period i. The general pricing rule for given capacity is given by equation (69), and the general optimal capacity rule is given by (70).

Specifically, when $x_1 = Q$ and $x_2 < Q$ we have:

- In period 1 both plants are used at its maximum capacity.

$$x_{11} = Q_1, \qquad x_{21} = Q_2 \tag{72}$$

- In period 2 plants are used in merit order that is plant 1 will be used first. Assume plant 1 is used at its maximum capacity, while plant 2 is unused.

$$x_{12} = Q_1, \qquad x_{22} = 0 \tag{73}$$

From (71) we obtain the shadow prices for the capacity constraint:

$$\lambda_{11} > 0, \qquad \lambda_{12} > 0, \qquad \lambda_{21} > 0, \qquad \lambda_{22} = 0 \tag{74}$$

According to (70) optimal capacity for technology 1 is found where the marginal willingness to pay for technology 1 of both peak-and off-peak customer's is equal to the actual cost of installing one additional unit capacity unit of technology 1:

$$\beta_1 = \lambda_{11} + \lambda_{12} \tag{75}$$

Optimal capacity for technology 2 is found where the marginal willingness to pay for technology 2 of peak customer's is equal to the actual cost of installing one additional unit capacity unit of technology 2:

$$\beta_2 = \lambda_{21} \tag{76}$$

Studying (69) for the different combinations of $l = 1,2$ and $i = 1,2$ gives the following prices when capacity is optimized:

$$p_1 = b_2 + \beta_2 \,, \qquad p_2 = \beta_1 + 2b_1 - (b_2 + \beta_2) \tag{77}$$

4.5.3 Implication for the Steiner result

Result 1 may be valid with multiple technologies because some sort of marginal cost pricing is still relevant. Optimal peak price is equal to the marginal cost of expansion of the peak quantity, whereas optimal off-peak price is set equal to the marginal cost of expansion of the off-peak quantity (assumed that peak demand is also expanded in the same proportion). Price is set equal to the marginal cost of expanding the demand in that period.

Result 4 has to be extended in the diverse technology case because the capacity decision has to be understood in a broader integrated resource-planning context. Plants are to be installed and operated in order of increasing operating costs and decreasing capacity costs. Optimal capacity of a type of technology is found where the sum of the marginal willingness to pay for an additional unit of capacity over the periods is equal to the actual cost of increasing that capacity by one unit.

Result 3 is no longer valid when multiple technologies are considered. Off-peak users are made to contribute to capacity costs related to the base load capacity, β_1, because base load capacity is continuously operated and used by all consumers both in peak and off-peak periods. Plants are required to meet not only peak demand but also base demand, thus the additional capacity costs involving installing new base load plants must be born by all the consumers. This implies that off-peak customers also press against some capacity, and hence corresponding capacity costs are allocated to the off-peak consumers.

Weintraub (1970) argues that the Steiner one-technology solution is unacceptable on welfare grounds because of the so-called “free rider” problem. Off-peak consumers pay no capacity costs but are supplied output out of capacity. Crew and Kleindorfer (1975, p. 88) explain that the income distribution objections may be overcome by a more complex technology. The pricing rule with multiple technologies differs from the Steiner solution in that off-peak users are made to pay a fraction of the capacity costs in proportion to its relative total utilization, thus having favourable income distribution effects for the peak users.

4.6 Unequal length of periods

Williamson (1966; 1974) allow for unequal length of periods. Let $w_1$and w_2 be the fraction of a cycle accounted for by the peak period and off-peak period ($w_1 + w_2 = 1$). The maximization problem with unequal length of periods, i.e. $w_i \neq \frac{1}{2}$:

$$\max_{x_i,Q}\left[\sum_{i=1}^{2}\int_0^{x_i} p_i\,(x_i')dx_i'\,w_i - \sum_{i=1}^{2} bx_i\,w_i - \beta Q\right]$$

subject to

$$x_i \leq Q$$
$$p_i = p_i(x_i)$$
$$p_1(x) > p_2(x)$$
$$x_i, Q > 0$$
$$b, \beta, w_i\ given, i = 1{,}2$$

The Lagrangian to the problem is:

$$L(x_i,Q) = \sum_{i=1}^{2}\int_0^{x_i} p_i\,(x_i')dx_i'\,w_i - \sum_{i=1}^{2} bx_i\,w_i - \beta Q - \sum\nolimits_{i=1}^{2}\lambda_i\,(x_i - Q)$$

Endogenous variables are x_i, Q and λ_i ($i = 1{,}2$). The necessary first-order conditions are:

$$\frac{\partial L(x_i,Q)}{\partial x_i} = p_i w_i - bw_i - \lambda_i = 0$$
$$\frac{\partial L(x_i,Q)}{\partial Q} = -\beta + \sum\nolimits_{i=1}^{2}\lambda_i = 0$$
$$\lambda_i \geq 0, [= 0\ for\ x_i < Q]$$

The general pricing rule is:

$$p_i = b + \frac{\lambda_i}{w_i} \tag{78}$$

Specifically, when $x_1 = Q$ and $x_2 < Q$ optimal short-run prices are:

$$p_1 = b + \frac{\lambda_1}{w_1}\,, \qquad p_2 = b \tag{79}$$

The larger the fraction of a cycle accounted for by the peak load, the lower the peak price since capacity can be divided over a larger time-interval. Allowing for unequal length of periods does not change the Steiner results.

5 Findings

This paper has studied how robust the general Steiner result for optimal pricing and optimal capacity are to changes in the assumptions.

Result 1 of price equal to long run marginal cost is not robust to changes in the assumption of linear costs, fully variable capacity in the long run, dependent demand, the objective of maximizing welfare and only considering the long run planning point of view. They are all crucial assumptions for result 1. Still, prices depend on marginal cost, however not the long-run marginal cost Steiner advocate but the short-run marginal cost. The demand for electricity exhibits substantial variations over periods of time too short to permit capacity to be varied so as to keep price continuously equal to long-run marginal cost. At any point in time capacity at that time is fixed and the only decision at that time is of pricing optimality within the capacity constraint. The Steiner prices are actually prices for a future cycle when capacity is optimized.

When considering a breakeven welfare-maximizing social planner or a profit-maximizing social planner, prices also depend on elasticity of demand. To ensure the firm at least break eve, a breakeven constraint is imposed when non-linear costs or fixed capacity in the long run is the case. If the monopoly is regulated the prices is affected by the specific regulation. When dependent demand is considered, prices also depend on the cross-elasticity of demand. However, the relevance of altering the assumption of independent demand is an empirical question. In addition, dependent demand may violate the framework of partial equilibrium model and thus be outside the scope of this paper. With multiple technologies some sort of marginal cost pricing is still relevant. Price is set equal to the marginal cost of expanding the demand in that period.

Result 2 of peak price higher than off-peak price follows automatically in the standard model in which there is a welfare objective, the firm has constant returns to scale in production and where capacity is fully variable in the long-run view. In general, nothing as strong as result 2 can be stated when considering a profit-maximizing monopoly or a breakeven welfare-maximizing social planner (to ensure non-negative profits in the case of constant returns to scale or fixed capacity in the long run). Then prices depend on elasticity of demand and the pricing reversal phenomenon may occur depending on the parameters. Additionally, when we relax the assumption of independent demand, which may not be that reasonable to study, price will also depend on the cross-elasticity of demand and may contribute to pricing reversal. The assumptions of welfare-objective, linear costs, capacity fully variable in the long run and independent demand are crucial assumptions for result 2. Notice, however if they are all relaxed result 2 may still be valid depending on the specific parameters.

Result 3 of no responsibility for capacity cost imputed to those customers whose demand does not press upon capacity has been criticized on welfare grounds since off-peak customers are also served by the capacity even if they do not press against the capacity. The result is not valid for the short-run peak load problem, as capacity cost is only related to the long-run peak load problem. The one-technology (i.e. homogenous plant capacity) assumption is crucial for the result that peak users bear all of the capacity costs. When diverse technology is introduced off-peak customers are made to contribute to capacity costs, since they press against the capacity limit to the base-load capacity.

Result 4 of optimal capacity found where it is equal to peak demand when optimized is relatively simple, and therefore *does* survive the different extensions reviewed. However, this is strictly speaking not a result of the Steiner model – it is a relationship by definition. For all the extensions of the model, optimal capacity is equal to peak load demand due to the imputed capacity constraint. However, how to find *optimal* capacity in the extended models is different than Steiner advocates.

When capacity constraint is considered the analysis of optimal investment is extended due to the dual variable followed by the capacity constraints. Optimal capacity is obtained where the sum of marginal willingness to pay over the periods for one additional over unit of capacity equals the actual cost of one additional unit of capacity. For the breakeven welfare-maximizing social planner and the unregulated profit-maximizing monopoly, the sum of marginal willingness to pay over the periods for one additional unit of capacity equals the actual cost of one additional unit of capacity *and* the shadow price on the breakeven constraint. This implies reduced optimum size of capacity compared to the pure welfare-maximizing capacity. For the regulated profit-maximizing monopoly the sum of marginal willingness to pay over the periods for one additional over unit of capacity equals the actual cost of one additional unit of capacity *and* an interaction effect between the capacity cost and the regulatory constraint, implying that optimal capacity is larger than the unregulated profit-maximizing monopoly (whether is it larger than the welfare-maximizing capacity level depends on the parameters). For multiple technologies the capacity decision has to be understood in a broader integrated resource-planning context. Plants are to be installed and operated in order of increasing operating costs and decreasing capacity costs. Optimal capacity of a type of technology is found where the sum of the marginal willingness to pay for an additional unit of capacity over the periods is equal to the actual cost of increasing that capacity by one unit. The Steiner result 4 is strictly speaking not altered when the assumptions is relaxed, and is robust. However, when the above assumptions are relaxed the valuable new information is obtained.

The robust result of Steiner's peak-load pricing when relaxing the above-mentioned assumptions is to set one price in each pricing period in accordance with the pattern of demand and prices are closely tied to variation in the marginal cost of generating electricity. Optimal capacity is equal to peak demand.

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Appendices

A Social Science Citation Index – results

Citation search according to Social Science citation Index yielded the following chronological results (30/04-2014).

Papers	Times cited
Bye (1929)	6
Lewis (1941a)	18
Lewis (1941b)	1
Houthakker (1951)	40
Steiner (1957)	**167**
Hirshleifer (1958)	61
Boiteux (1960)	101
Williamson (1966)	163
Gabor (1966)	2
Buchanan (1966)	14
Turvey (1968)	26
Mohring (1970)	102
Pressman (1970)	48
Littlechild (1970)	26
Weintraub (1970)	3
Crew and Kleindorfer (1971)	10
Bailey (1972)	24
Bailey and White (1974)	26
Williamson (1974)	0
Crew and Kleindorfer (1975)	6
Gravelle (1976)	7
Crew and Kleindorfer (1976)	67
Nguyen (1976)	7
Wenders (1976)	56
Panzar (1976)	42
Asbury and Mueller (1978)	-

Dansby (1978)	5
Crew and Kleindorfer (1981)	5
Lioukas (1983)	2
Gallant and Koenker (1984)	9
Gerstner (1986)	5
Bergstrom and MacKie-Mason (1991)	9
Kleindorfer and Fernando (1993)	24
Crew et al (1995)	79

B Mathematical calculations

B1

Multiply $\frac{\partial p_i(x_i)}{\partial x_i} x_i$ with $\frac{p_i(x_i)}{p_i(x_i)}$ then first order equation (22) is:

$$p_i - b - \lambda_i + \gamma\left(p_i(x_i) + \frac{\partial p_i(x_i)}{\partial x_i}\frac{x_i}{p_i(x_i)}p_i(x_i) - b\right) = 0$$

Since $\varepsilon_i = -\frac{p_i}{x_i}\frac{dx_i}{dp_i}$, we have:

$$p_i - b - \lambda_i + \gamma\left(p_i - \frac{p_i}{\varepsilon_i} - b\right) = 0$$

Rearranging we get:

$$p_i = \frac{\lambda_i + b(1+\gamma)}{1 + \gamma - \gamma\left(\frac{1}{\varepsilon_i}\right)}$$

If multiplied with $\frac{1+\gamma}{1+\gamma}$ we obtain equation (25).

B2

Because $i = 1,2$ and $j = 1,2$ first order equation (41) may be rewritten as:

$$p_i - b - \lambda_i + \gamma\left(p_i + \frac{\partial p_j}{\partial x_i}x_j + \frac{\partial p_i}{\partial x_i}x_i - b\right) = 0$$

Multiply$\frac{\partial p_j}{\partial x_i}x_j$ and $\frac{\partial p_i}{\partial x_i}x_i$ with $\frac{p_i}{p_i}$ then:

$$p_i - b - \lambda_i + \gamma\left(p_i + \frac{\partial p_j}{\partial x_i}\frac{x_j}{p_i}p_i + \frac{\partial p_i}{\partial x_i}\frac{x_i}{p_i}p_i - b\right) = 0$$

Using condition (38) of $\frac{\partial p_j}{\partial x_i} = \frac{\partial p_i}{\partial x_j}$ we have:

$$p_i - b - \lambda_i + \gamma\left(p_i + \frac{\partial p_i}{\partial x_j}\frac{x_j}{p_i}p_i + \frac{\partial p_i}{\partial x_i}\frac{x_i}{p_i}p_i - b\right) = 0$$

Since $\varepsilon_i = -\frac{p_i}{x_i}\frac{dx_i}{dp_i}$ and $\varepsilon_{ji} = -\frac{\partial x_i}{\partial p_j}\frac{p_j}{x_i}$:

$$p_i - b - \lambda_i + \gamma\left(p_i - \frac{p_i}{\varepsilon_{ji}} - \frac{p_i}{\varepsilon_i} - b\right) = 0$$

Rearranging we get:

$$p_i = \frac{\lambda_i + b(1+\gamma)}{1 + \gamma - \gamma\left(\frac{1}{\varepsilon_i} + \frac{1}{\varepsilon_{ji}}\right)}$$

If multiplied with $\frac{1+\gamma}{1+\gamma}$ we obtain equation (45).